An Expressive Hybrid Model for the Composition of Cardinal Directions

Ah Lian Kor and Brandon Bennett
School of Computing, University of Leeds, Leeds LS2 9JT, UK
e-mail: {Lian,Brandon}@Comp.leeds.ac.uk

Abstract
In our previous paper (Kor and Bennett, 2003), we have shown how the nine tiles in the projection-based model for cardinal directions can be partitioned into sets based on horizontal and vertical constraints (called Horizontal and Vertical Constraints Model). In order to come up with an expressive hybrid model for direction relations between two-dimensional single-piece regions (without holes), we integrate the well-known RCC-8 model with the above-mentioned model. From this expressive hybrid model, we derive 8 atomic binary relations and 13 model with the above-mentioned model. From this expressive and weak direction relations between 'whole or part' regions. Lastly, we also show how the expressive hybrid model can be used to make several existential inferences that are not possible for existing models.

Introduction
Papadias and Theodoris (1994) describe topological and direction relations between regions using their minimum bounding rectangles (MBRs). However, the language used is not expressive enough to describe the direction relations. Additionally, the MBR technique yields erroneous outcome when the involving regions are not rectangular in shape (Goyal and Egenhofer, 2000). In order to come up with a more expressive language for direction relations, we shall combine mereological, topological, and cardinal directions relations.

Typically, composition tables are used to infer spatial relations between objects. However, existing composition tables for cardinal relations(Escrig and Toledo, 1998; Skiodopoulos and Koubarakis, 2000) are weak and not expressive enough. Consequently, these tables cannot make some existential inferences that will be shown at the later part of this paper. Here, we shall show how we come up with an expressive hybrid model for direction relations. Based on this model, we derive two 8x8 composition tables for expressive as well as weak direction relations.

In this paper, we shall describe the binary relations in the expressive hybrid model for direction relations, and define 'whole and part' relations. This is followed by introducing a formula which could be used to compute both expressive and weak direction relations for ‘whole and part’ regions. Finally, we shall demonstrate how the model could be used to make several types of existential inferences.

Horizontal and Vertical Constraints Model
In the projection-based model for cardinal directions (Frank, 1992), the plane of an arbitrary single-piece region \( a \), is partitioned into nine tiles, North-West, NW(\( a \)); North, N(\( a \)); North-East, NE(\( a \)); South-West, SW(\( a \)); South, S(\( a \)); South-East, SE(\( a \)); West, W(\( a \)); Neutral Zone, O(\( a \)); East, E(\( a \)). In our previous paper (Kor and Bennett, 2003), we have shown how to partition the nine tiles into sets based on horizontal and vertical constraints called the Horizontal and Vertical Constraints Model. However, in this paper, we shall rename the sets for easy comprehension purposes. The following are the definitions of the partitioned regions:

- **WeakNorth(\( a \))** is the region that covers the tiles NW(\( a \)), N(\( a \)), and NE(\( a \))
- **Horizontal(\( a \))** is the region that covers the tiles W(\( a \)), O(\( a \)), and E(\( a \))
- **WeakSouth(\( a \))** is the region that covers the tiles SW(\( a \)), S(\( a \)), and SE(\( a \))
- **WeakWest(\( a \))** is the region that covers the tiles SW(\( a \)), W(\( a \)), and NW(\( a \))
- **Vertical(\( a \))** is the region that covers the tiles S(\( a \)), O(\( a \)), and N(\( a \))
- **WeakEast(\( a \))** is the region that covers the tiles SE(\( a \)), E(\( a \)), and NE(\( a \))

The set of boundaries of the minimal bounding box for region \( a \) is could be represented as \([X_{ns}(a), X_{ne}(a), Y_{se}(a), Y_{sn}(a)]\). Figure 1 depicts the horizontal and vertical sets of tiles for \( a \).

![Figure 1: Horizontal and Vertical Sets of Tiles](image)

RCC-8 Model
The RCC-8 (Randell et al, 1992) model consists of 8 atomic binary relations: PO(\( a,b \)), TPP(\( a,b \)), NTPP(\( a,b \)),...
Expressive Hybrid Model

In order to come up with a more expressive model for the composition of cardinal directions, we integrate the RCC-8 and the Horizontal and Vertical Constraints Model. We shall consider two-dimensional single-piece regions without hole. We shall consider the x and y dimensions separately.

Definitions

If there is a referent region a, and another arbitrary region b, the possible atomic binary relations between them can be defined as follows:

- **WeakNorth(b,a)** - b \(\subseteq\) **WeakNorth(a)**
- **Horizontal(b,a)** - b \(\subseteq\) **Horizontal(a)**
- **WeakSouth(b,a)** - b \(\subseteq\) **WeakSouth(a)**
- **WeakEast(b,a)** - b \(\subseteq\) **WeakEast(a)**
- **Vertical(b,a)** - b \(\subseteq\) **Vertical(a)**
- **WeakWest(b,a)** - b \(\subseteq\) **WeakWest(a)**
- **DCy(a,b)** - y-dimension of a is disconnected from y-dimension of b
- **EQy(a,b)** - y-dimension of a is identical with y-dimension of b
- **POy(a,b)** - y-dimension of a partially overlaps y-dimension of b
- **ECx(a,b)** - y-dimension of a is externally connected to x-dimension of b
- **TPPy(b,a)** - y-dimension of a is a tangential proper part of y-dimension of b
- **NTPPy(b,a)** - y-dimension of a is a nontangential proper part of y-dimension of b
- **TPPb(a,b)** - y-dimension of a is a tangential proper part of y-dimension of a
- **NTPPb(a,b)** - y-dimension of b is a nontangential proper part of y-dimension of a

Atomic Binary Relations of the Hybrid Model

In this section, we shall demonstrate how we come up with all possible binary direction relations for the hybrid model. All the possible atomic binary relations for each horizontal set are shown in Figure 2. The notations that will be used in this section are:

- **RELx(b,Z)** is any atomic binary relation between b and the horizontally partitioned region, Z
- **RELx(b,Z)** is any atomic binary relation between b and the vertically partitioned region, Z

Based on Table 1, the total number of possible binary relations for the hybrid model in the y-direction is \((24+4+2) + (24+2) + (4x2) + (2x4x2)\) which equals 44 cases. However, due to the single-piece condition, the following rules apply:

- **Rule 1:** \(b \subseteq \neg(\text{WeakNorth}(a))\)
- **Rule 2:** Assume \(U\) to be \{WeakNorth(a), Horizontal(a), WeakSouth(a)\}. If \(\neg\text{NTPPb}(b,R)\) where \(R \in U\) then \(\neg(\text{NTPPb}(b,R) \land \text{RELx}(b,S)) \lor \neg(\text{NTPPb}(b,R) \land \text{RELx}(b,T))\) where \(S \in U \cdot R, T \in U \cdot S\).

Based on the rules above, the total number of feasible binary relations for single-piece regions in the y-direction is \((44 - 4 - 23 - 4)\) which equals 13 cases. The thirteen feasible and jointly exhaustive binary relations for the hybrid model are depicted in Figure 3. This means that in the hybrid model, the number of jointly exhaustive binary relations (in both the x and y directions) that hold between two single-piece regions will be 13x13. This concurs with the 13x13 atomic relations in the Rectangle Algebra Model (Balbiani et al. 1998).

Combined Mereological, Topological and Cardinal Direction Relations

In this section, we shall make two distinctions: ‘whole and part’ cardinal directions, as well as ‘weak and expressive’ relations. We shall rewrite the notations used in our previous paper (Kor and Bennett, 2003). \(P(b,a)\) means that only part of the destination extended region, b, is in tile R(a). The direction relation \(A(b,a)\) means that whole destination extended region, b, is in the tile R(a).

As an example, when b is completely in the South-East tile of a, this direction relation can be represented as below:

\[
A_{SE}(b,a) = \neg P_{SE}(b,a) \land \neg P_{SW}(b,a) \land \neg P_{EW}(b,a) \land \neg P_{NW}(b,a) \land \neg P_{W}(b,a) \land \neg P_{E}(b,a) \land \neg P_{S}(b,a) \land \neg P_{N}(b,a)
\]

The ‘whole and weak’ direction relations are defined in terms of horizontal and vertical sets.

- **A_{W}(b,a) = WeakNorth(b,a) \land Vertical(b,a)**
- **A_{W}(b,a) = WeakNorth(b,a) \land WeakEast(b,a)**
- **A_{W}(b,a) = WeakNorth(b,a) \land WeakWest(b,a)**
- **A_{W}(b,a) = WeakSouth(b,a) \land Vertical(b,a)**
- **A_{W}(b,a) = WeakSouth(b,a) \land WeakEast(b,a)**
- **A_{W}(b,a) = WeakSouth(b,a) \land WeakWest(b,a)**
- **A_{W}(b,a) = Horizontal(b,a) \land Vertical(b,a)**
- **A_{W}(b,a) = Horizontal(b,a) \land WeakEast(b,a)**
- **A_{W}(b,a) = Horizontal(b,a) \land WeakWest(b,a)**

The ‘whole and expressive’ direction relations are defined in terms of expressive horizontal and vertical sets. A general form of such direction relation can be represented as follows:

\[
\forall(b,a) 
\begin{array}{l}
\text{REL}_{H}(b,a) \equiv \text{RELx}(b,H) \land \text{RELx}(b,V) \\
\text{REL}_{V}(b,a) \equiv \text{REL}(b,H) \land \text{RELx}(b,V)
\end{array}
\]
### Composition of Regions with Parts

In our previous paper (Kor and Bennett, 2003), the method for the computation of part regions is not robust enough because it does not hold for all cases. In order to address such a problem, we introduce a formula (Equation 2.a). The basis of the formula is to consider the direction relation between each individual part of \( a \) and each individual part of \( b \) followed by the direction relation between each individual part of \( b \) and \( c \). Assume that the region \( b \) covers one or more than one tile of region \( a \) while region \( c \) encompasses one or more than one tile of region \( b \). The formula for the composition of weak direction relations can be written as follows:

\[
P_f(b,a) \land P_f(c,b) = \left[ P_{ed}(b,a) \land P_{ed}(b,a) \ldots \land P_{ed}(b,a) \right] \land \left[ P_{ed}(a,c) \right]
\]

\( 1 \leq k \leq 9 \), and \( 1 \leq m \leq 9 \)

Consider the composition of direction for each individual part of \( b \) and \( c \). Equation (1) becomes:
Use Equation Type 2:

Find the composition of

Firstly, we shall demonstrate how to apply the formula

This means that the region

The outcome of the composition is:

By applying distributive law we have the following equation:

Therefore, the above composition can be rewritten as:

Both \( c_i \subseteq c \) and \( c_i \subseteq c \), so the above outcome can be written as:

This means that the region \( c \subseteq E(a) \lor O(a) \lor W(a) \lor NE(a) \lor N(a) \lor NW(a) \).

Type 3: \( P_\alpha(b,a) \land A_\beta(c,b) \)

Find the composition of \( [P_\alpha(b,a) \land P_\beta(b,a)] \land A_\gamma(c,b) \)

Establish the relationship between \( c \) and each individual part of \( b \). In this case, when \( A_\gamma(c,b) \), \( P_\beta(b,c) \) and \( P_\alpha(b,c) \) holds (this is not necessarily true for all cases).

Use Equation \( 2.a \) with \( 1 \leq k \leq 2 \) and \( m = 1 \).

The outcome of the composition is:

Type 2: \( A_\alpha(b,a) \land P_\beta(c,b) \)

Find the composition of \( A_\alpha(b,a) \land [P_\beta(c,b) \land P_\gamma(c,b)] \)

Use Equation \( 2.a \) with \( k = 1 \), and \( 1 \leq m \leq 2 \).

This means that the region \( c \subseteq O(a) \lor W(a) \lor S(a) \lor SW(a) \).

The outcome of the composition is:

The outcome of the composition is:

This means that the \( Y_m(c) \) of the minimal bounding box for region \( c \) is greater than \( Y_m(a) \) of the minimal bounding box for region \( a \) and \( c \subseteq NE(a) \lor N(a) \).
**Type 4: \( P_r(b,a) \land P_s(c,b) \)**

Find the composition of 
\[
[P_d(b,a) \land P_RW(c,b)] \land [P_d(c,b) \land P_s(c,b) \land P_SZ(c,b)]
\]

The diagram Figure 4 has been drawn for this example.

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![Diagram](image)

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**Figure 4: An example**

Establish the direction relation between each individual part of \( b \) and \( c \).

Use Equation 2.a with \( 1 \leq k \leq 2 \), the value of \( m_j \) for \( b \) is \( 1 \leq m_j \leq 4 \).  
While the value of \( m_j \) for \( b \), is \( 1 \leq m_j \leq 7 \). 
\[
[P_d(b,a) \land P_RW(c,b)] \lor [P_d(b,a) \land P_RW(c,b)] \land [P_d(b,a) \land P_RW(c,b)] \land [P_d(b,a) \land P_RW(c,b)] 
\]

\[
[P_d(b,a) \land P_RW(c,b)] \land [P_d(b,a) \land P_RW(c,b)] \land [P_d(b,a) \land P_RW(c,b)] \land [P_d(b,a) \land P_RW(c,b)] 
\]

In part(2) of composition, \( c_1, c_2, c_3, c_4, c_5, c_6, c_7, c \in c \). The simplified version of the composition is as follows: 
\[
[P_d(b,a) \land P_RW(c,b)] \land [P_d(b,a) \land P_RW(c,b)] \land [P_d(b,a) \land P_RW(c,b)] 
\]

This means that the region \( c \subseteq U \) which is the union of all 9 tiles of region \( a \). However, based on Figure 4, region \( c \subseteq SW(a) \).

**Composition of Expressive Direction Relations**

We shall use the following notations to represent the 13 binary \( y \)-direction relations:

- REL1y(b,a)–TPPy(b,WeakNorth(a))
- REL2y(b,a)–TPPy(b,WeakNorth(a))–ECy(b,Horizontal(a))
- REL3y(b,a)–TPPy(b,Horizontal(a))–ECy(b,WeakNorth(a))
- REL4y(b,a)–TPPy(b,Horizontal(a))–ECy(b,WeakSouth(a))
- REL5y(b,a)–NTPPy(b,Horizontal(a))
- REL6y(b,a)–EQy(b,Horizontal(a))
- REL7y(b,a)–NTPPy(b,WeakSouth(a))
- REL8y(b,a)–TPPy(b,WeakSouth(a))–ECy(b,Horizontal(a))
- REL9y(b,a)–POy(b,WeakNorth(a))–POy(b,Horizontal(a))–DCy(b,WeakSouth(a))
- REL10y(b,a)–POy(b,WeakNorth(a))–POy(b,Horizontal(a))–DCy(b,WeakSouth(a))
- REL11y(b,a)–POy(b,WeakNorth(a))–POy(b,Horizontal(a))–NTPPy(b,Horizontal(a))
- REL12y(b,a)–POy(b,WeakSouth(a))–POy(b,Horizontal(a))–NTPPy(b,Horizontal(a))
- REL13y(b,a)–POy(b,WeakSouth(a))–POy(b,Horizontal(a))–ECy(b,WeakNorth(a))

Similar notations will be used to represent the 13 binary \( x \)-direction relations (WeakNorth is replaced by WeakEast, Horizontal with Vertical and WeakSouth by WeakWest).
Example 1:
Find the composition of the following:
\[
\left[\text{Equation 1}\right] \equiv \left[\text{Equation 2}\right]
\]
Establish the direction relation between \(c\) and each individual part of \(b\). Use Equation 2, with \(1 \leq k \leq 2\) and \(1 \leq m \leq 2\).
\[
\left[\text{Equation 3}\right] \equiv \left[\text{Equation 4}\right]
\]
Use Equation (1), and the above composition can be written as follows:
\[
\left[\text{Equation 5}\right] \equiv \left[\text{Equation 6}\right]
\]
This example is similar to the fourth example in the previous section of this paper.
For part \(b\),
\[
\left[\text{Equation 7}\right] \equiv \left[\text{Equation 8}\right]
\]
Existential Inference

In this paper, we shall demonstrate how the expressive hybrid cardinal direction model could be used to make several existential inferences which are not possible in existing models.

Example 1: Find \( R(b,a) \) such that \( c \subset \text{WeakNorth}(b) \) and \( c \subset \text{WeakNorth}(a) \)

Skiadopoulos model (2001) which is not expressive enough, cannot answer such query because the composition table computed is not existential. To answer this query, we must first specify the expressive relation between \( a \) and \( c \).

There are two possible relations: \( \text{TPPy}(c,\text{WeakNorth}(a)) \) or \( \text{WeakNorth}(c,a) \). If it is the former then composition is \( \text{R(b,a)} \wedge \text{WeakNorth}(c,b) \) which means \( R(b,a) \) is \( \text{WeakNorth}(b,a) \). If it is the latter, there are several combinations:

- \( \text{WeakNorth}(b,a) \wedge \text{Horizontal}(c,b) \)
- \( \text{WeakNorth}(b,a) \wedge \text{WeakSouth}(c,b) \)
- \( \text{Horizontal}(b,a) \wedge \text{WeakNorth}(c,b) \)
- \( \text{WeakSouth}(b,a) \wedge \text{WeakNorth}(c,b) \)

The first relation in each composition listed above is \( R(b,a) \).

Example 2: Find \( R(b,a) \) and \( S(c,b) \) such that \( T(a,c) \) is \( \neg [\text{TPPy}(c,\text{Horizontal}(a)) \wedge \text{ECy}(c,\text{WeakSouth}(a))] \)

Based on Table 1, 9 different compositions will yield the following outcome:

\[
\text{TPPy}(c,\text{Horizontal}(a)) \wedge \text{ECy}(c,\text{WeakSouth}(a))
\]

The set of possible compositions, \( Q \), is:

\[
\{ \text{REL1y}(b,a) \wedge \text{REL7y}(c,b), \text{REL2y}(b,a) \wedge \text{REL7y}(c,b), \text{REL3y}(b,a) \wedge \text{REL8y}(c,b), \text{REL5y}(b,a) \wedge \text{REL7y}(c,b), \text{REL6y}(b,a) \wedge \text{REL3y}(c,b), \text{REL7y}(b,a) \wedge \text{REL3y}(c,b), \text{REL8y}(b,a) \wedge \text{REL2y}(c,b) \}
\]

If \( U \) equals 8 x 8 atomic binary direction relations, then the set of all possible ordered pairs of \( R \) and \( S \) which satisfy the above query will be \( U - Q \).

Example 3: Find \( R(b,a) \) and \( S(c,b) \) such that \( T(a,c) \) is \( \text{POy}(c,\text{WeakSouth}(a)) \wedge \text{POy}(c,\text{Horizontal}(a)) \wedge \text{ECy}(c,\text{WeakNorth}(a)) \)

Based on Table 1, we have 4 pairs of \( R \) and \( S \) which satisfy \( T \). They are: \( \text{REL1y}(b,a) \wedge \text{REL7y}(c,b), \text{REL2y}(b,a) \wedge \text{REL8y}(c,b), \text{REL7y}(b,a) \wedge \text{REL1y}(c,b), \text{REL7y}(b,a) \wedge \text{REL2y}(c,b) \).

Conclusion

In this paper, we have shown how topological and direction relations can be integrated to produce a more expressive hybrid model for cardinal directions. The composition table derived from this model could be used to infer both weak and expressive direction relations between regions. We have also introduced and demonstrated how to use a formula to compute the composition of weak or expressive relations between ‘whole and part’ regions. We have also demonstrated how the composition table with expressive direction relations could be used to make several difficult existential inferences.

Acknowledgement

Many thanks to Syhamanta Hazarika for his valuable comments.

References


<table>
<thead>
<tr>
<th>WeakNorth(a)</th>
<th>Horizontal(b)</th>
<th>WeakSouth(b)</th>
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<td>NTPPy(c, WeakNorth(a)) ∨ Horizontal(c, WeakNorth(a)) ∨ TPY(c, WeakNorth(a)) ∨ POy(c, WeakNorth(a))</td>
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</tr>
</tbody>
</table>

U = 13 relations

NTPPy(c, WeakNorth(a)) ∨ Horizontal(c, WeakNorth(a)) ∨ TPY(c, WeakNorth(a)) ∨ POy(c, WeakNorth(a)) ∨ POy(a, Horizontal(a)) ∨ DCy(c, WeakNorth(a))

NTPPy(c, WeakSouth(b)) ∨ Horizontal(c, WeakSouth(b)) ∨ TPY(c, WeakSouth(b)) ∨ POy(c, WeakSouth(b)) ∨ POy(a, Horizontal(a)) ∨ DCy(c, WeakSouth(b))
**Note:** The shaded boxes are for weak composition of relations.

<table>
<thead>
<tr>
<th>WeakNorth(a)</th>
<th>Horizontal(a)</th>
<th>WeakSouth(a)</th>
<th>TPy(b, WeakSouth(a)) &amp; ECy(b, Horizontal(a))</th>
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**Table 1:** Composition of binary relations in the y-direction for the hybrid model
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<td>TPPx(b, WeakEast(a)) \text{∧} \text{ECx(b, Vertical(a))}</td>
<td>NTPx(c, WeakEast(a))</td>
<td>NTPx(c, WeakEast(a))</td>
<td>NTPx(c, WeakEast(a))</td>
<td>NTPx(c, WeakEast(a))</td>
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<td>NTPx(c, WeakEast(a))</td>
<td>NTPx(c, WeakEast(a))</td>
<td>\text{↔} \text{ECx(c, Vertical(a))}</td>
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<tr>
<td>TPPx(b, Vertical(a)) \text{¬} \text{ECx(b, WeakEast(a))}</td>
<td>NTPx(c, WeakEast(a))</td>
<td>NTPx(c, WeakEast(a))</td>
<td>NTPx(c, WeakEast(a))</td>
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<td>NTPx(c, WeakEast(a))</td>
<td>NTPx(c, WeakEast(a))</td>
<td>TPPx(c, Vertical(a)) \text{¬} \text{ECx(c, WeakWest(a))}</td>
</tr>
<tr>
<td>TPPx(b, Vertical(a)) \text{¬} \text{ECx(b, WeakWest(a))}</td>
<td>NTPx(c, Vertical(a)) \text{¬} \text{ECx(c, Vertical(a))}</td>
<td>NTPx(c, Vertical(a)) \text{¬} \text{ECx(c, WeakEast(a))}</td>
<td>NTPx(c, Vertical(a)) \text{¬} \text{ECx(c, WeakWest(a))}</td>
<td>NTPx(c, Vertical(a)) \text{¬} \text{ECx(c, WeakEast(a))}</td>
<td>NTPx(c, Vertical(a)) \text{¬} \text{ECx(c, WeakWest(a))}</td>
<td>NTPx(c, Vertical(a)) \text{¬} \text{ECx(c, WeakEast(a))}</td>
<td>NTPx(c, Vertical(a)) \text{¬} \text{ECx(c, WeakWest(a))}</td>
<td>\Rightarrow \text{POx(c, WeakWest(a)) \text{¬} \text{POx(a, Vertical(a)) \text{¬} \text{DCy(c, WeakEast(a))}}}</td>
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<br>
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<tr>
<th>WeakEast(a)</th>
<th>Vertical(b)</th>
<th>WeakWest(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NTPPx(c, WeakEast(b))</strong></td>
<td><strong>TPPx(c, Vertical(b))</strong></td>
<td><strong>NTPPx(c, WeakWest(b))</strong></td>
</tr>
<tr>
<td>▲ <strong>ECx(c, Vertical(b))</strong></td>
<td>▲ <strong>ECx(c, WeakEast(b))</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Vertical(b) = Vertical(a)</strong></td>
<td><strong>NTPPx(c, Vertical(a))</strong></td>
<td><strong>NTPPx(c, Vertical(a))</strong></td>
</tr>
<tr>
<td>▲ <strong>TPPx(c, Vertical(a))</strong></td>
<td>▲ <strong>TPPx(c, Vertical(a))</strong></td>
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<tr>
<td>▲ <strong>NTPPx(c, Vertical(a))</strong></td>
<td>▲ <strong>NTPPx(c, Vertical(a))</strong></td>
<td></td>
</tr>
<tr>
<td><strong>NTPPx(b, Vertical(a))</strong></td>
<td><strong>TPPx(b, WeakWest(a))</strong></td>
<td><strong>NTPPx(b, WeakWest(a))</strong></td>
</tr>
<tr>
<td>▲ <strong>ECx(b, Vertical(a))</strong></td>
<td>▲ <strong>ECx(b, Vertical(a))</strong></td>
<td></td>
</tr>
<tr>
<td><strong>TPPx(b, WeakWest(a))</strong></td>
<td><strong>NTPPx(b, WeakWest(a))</strong></td>
<td><strong>NTPPx(b, Vertical(a))</strong></td>
</tr>
<tr>
<td>▲ <strong>EQx(b, Vertical(a))</strong></td>
<td>▲ <strong>EQx(b, Vertical(a))</strong></td>
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</tr>
<tr>
<td>▲ <strong>NTPPx(b, Vertical(a))</strong></td>
<td>▲ <strong>NTPPx(b, Vertical(a))</strong></td>
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</tr>
<tr>
<td><strong>WeakEast(a)</strong></td>
<td><strong>Vertical(b) = Vertical(a)</strong></td>
<td><strong>WeakWest(a)</strong></td>
</tr>
<tr>
<td><strong>U – 13 relations</strong></td>
<td><strong>NTPPx(c, WeakEast(a))</strong></td>
<td><strong>NTPPx(c, WeakWest(a))</strong></td>
</tr>
<tr>
<td>▲ <strong>TPPx(c, WeakEast(a))</strong></td>
<td>▲ <strong>TPPx(c, WeakWest(a))</strong></td>
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<tr>
<td>▲ <strong>NTPPx(c, WeakEast(a))</strong></td>
<td>▲ <strong>NTPPx(c, WeakWest(a))</strong></td>
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</tr>
<tr>
<td><strong>TPPx(b, WeakEast(a)) &amp; ECx(b, Vertical(a))</strong></td>
<td>▲ <strong>TPPx(c, WeakEast(b))</strong></td>
<td>▲ <strong>TPPx(c, WeakWest(b))</strong></td>
</tr>
<tr>
<td>▲ <strong>EQx(c, Vertical(a))</strong></td>
<td>▲ <strong>EQx(c, Vertical(a))</strong></td>
<td></td>
</tr>
<tr>
<td>▲ <strong>NTPPx(c, WeakEast(b))</strong></td>
<td>▲ <strong>NTPPx(c, WeakWest(b))</strong></td>
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<tr>
<td>▲ <strong>NTPPx(c, WeakEast(b))</strong></td>
<td>▲ <strong>NTPPx(c, WeakWest(b))</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The shaded boxes are for weak composition of relations.

Table 2: Composition of binary relations in the x-direction for the hybrid model.