Cooperative Optimal Preview Tracking for Linear Descriptor Multi-Agent Systems

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Abstract In this paper, a cooperative optimal preview tracking problem is considered for continuous-time descriptor multi-agent systems with a directed topology containing a spanning tree. By the acyclic assumption and state augmentation technique, it is shown that the cooperative tracking problem is equivalent to local optimal regulation problems of a set of low-dimensional descriptor augmented subsystems. To design distributed optimal preview controllers, restricted system equivalent (r.s.e.) and preview control theory are first exploited to obtain optimal preview controllers for reduced-order normal subsystems. Then, by using the invertibility of restricted equivalent relations, a constructive method for designing distributed controller is presented which also yields an explicit admissible solution for the generalized algebraic Riccati equation. Sufficient conditions for achieving global cooperative preview tracking are proposed proving that the distributed controllers are able to stabilize the descriptor augmented subsystems asymptotically. Finally, the validity of the theoretical results is illustrated via numerical simulation.

Keyword Descriptor multi-agent systems; cooperative tracking; optimal preview control; distributed control

1. Introduction

Recently, many researchers are increasingly interested in cooperative control of multi-agent systems, largely because of extensive applications of this subject in various fields, such as flocking behavior of animals [1], formation control of spacecraft [2], sensor networks [3], and so forth. As one of the most fundamental issues in this field, consensus plays an important role in reaching collective behavior. The crucial idea of consensus is to design a distributed control scheme, depending on local neighbor information, to drive all the agents to reach an agreement on some variables of interest. From this perspective, a number of cooperative control problems, such as swarm [4], flocking [5] and containment control [6], can be essentially viewed as consensus problems.

In most previous references about cooperative control problems for multi-agent systems, stability is mainly discussed. Optimality is also a primary property, and the control protocols with optimality can give rise to desirable properties, e.g., robust stability. In [7], the authors considered optimal consensus problem for multi-agent systems with single-integrator dynamics based on linear quadratic regulator (LQR). However, the optimal control proposed in [7] is not distributed. In [8], the authors applied inverse optimality and partial stability theorems to devise distributed protocols that can guarantee global optimal regulator performance, and provided sufficient conditions for the existence of these protocols. Compared with [8], necessary and sufficient conditions were established for global optimal cooperative control in [9], where the resulting optimal distributed protocols can make multi-agent systems reach desirable convergence rate and damping rate asymptotically. In [10], the authors brought together cooperative control, game theory and reinforcement learning to formulate the notion of differential graphical games for multi-agent systems, and designed a policy iteration algorithm on the basis of local information. More recently, the problem proposed in [9] was considered in [11], and the approach of [10] was extended to address distributed optimal tracking problems [12,13].

Descriptor systems (also known as singular systems, semi-state systems or differential algebraic systems) are natural representations of the real world. On account of their special features, such as regularity and impulse mode [14-16, 38], consensus problems for descriptor multi-agent systems are more complicated and challenging than those of normal multi-agent systems. The admissible consensus problems for descriptor multi-agent systems were discussed in [17-19], where static output feedback and dynamical output feedback were utilized in [17] and [18], respectively, and the scenario of descriptor heterogeneous multi-agent systems was treated in [19]. By resorting to feedforward control technique and reduced-order design, the authors of [20] settled the cooperative output regulation problem of singular heterogenous multi-agent systems. By employing observability decomposition and eigenvalue decomposition techniques, the authors of [21] investigated the admissible output consensus problems for singular swarm systems. In [22], the authors considered the guaranteed-cost consensus problems for descriptor multi-agent systems under switching topologies, and provided a method for achieving a trade-off between consensus regulation performance and control energy consumption. In [23], the authors converted containment problems for singular swarm systems with time delays into stability problems of a set of low-dimensional time-delayed systems, and presented sufficient conditions for containment control via linear matrix inequality.

In design of control systems, if whole or parts of the reference signals or (and) disturbance signals are known in advance, then the advanced information can be exploited to improve the system performance. Such a control system, for which the future information is available, is usually designated as preview control system. In [24-25], state augmentation technology and LQR theory were applied to optimal preview tracking problem in discrete and continuous time settings, respectively. Based on [25], the authors of [26] considered the scenario that reference signals and disturbance signals are previewable simultaneously. However, there exist a few inadequacies for LQR control in tackling optimal preview tracking problem. To this end, some researchers considered the preview tracking problem from the perspective of H_2 and H_∞ optimal control. Viewing the reference signal as an additional perturbation, the authors of [27] adopted differential game approach to study the H_∞ optimal preview tracking problem for continuous time-varying systems, and derived necessary and sufficient conditions for saddle-point equilibrium. In [28], the authors extended the approach of [27] to the discrete-time case. The authors of [29] considered the H_∞ preview full-information control problem, whose analytic solution was obtained by an auxiliary matrix Riccati equation. H_2 preview control problem was studied in [30] and [31], where the main difference is that the method of [31] can be utilized to treat multiple inputs and multiple preview times cases.

For linear descriptor system, the optimal preview control problem was studied in [32]. For multi-agent systems, the cooperative optimal preview tracking problem was first considered in [33]. It is worth pointing out that the controller in [32] was designed based on the Second Equivalent Form, and the controller in [33] was not distributed. Consequently, the following problems arise naturally: through the invertibility of restricted equivalent relations, can the optimal preview controller be derived constructively with respect to the matrices of original system? In addition, how can we devise distributed optimal preview controllers according to communication topology among agents? Motivated by [13], as well as the above questions, the current paper solves the cooperative optimal preview tracking problem for impulsive-free descriptor multi-agent systems.

Compared with the results in [32-33], this paper has the following four features. Firstly, the cooperative preview tracking problem considered in [33] is only a special case of the current paper. Secondly, unlike the centralized control method used in [33], the distributed design method is provided in this paper, which has the capability to decrease the computation complexity. Moreover, it is observed from the structure of the controller that preview compensation is only necessary for a small subset of followers that can receive information from the leader directly, but all the followers would achieve global cooperative preview tracking by using the distributed optimal controllers. Thirdly, the current paper develops the results in [32] from two aspects. One is the associated results for reaching preview tracking, which are given based on the matrices of descriptor system. The other is the numerical simulation that is performed by establishing a trapezoidal iteration scheme from original system. Last but by no means least, the distributed design approach is valid for descriptor multi-agent systems that are heterogenous, and also whose kinetics are impulse controllability and impulse observability.

Throughout this paper, $R^{n\times m}$ and $C^{n\times m}$ are the sets of $n\times m$ real and complex matrices, respectively; $\bar{C}^{+}$ denotes the closed right half complex plane; $diag(A_1, A_2, \cdots, A_n)$ denotes a diagonal matrix with submatrices A_i, $i=1,2,\cdots,n$ on its diagonal and zero elsewhere; $A\otimes B$ represents the Kronecker product of matrix

A and B; $\mathbf{1}_n \in R^n$ is the column vector with all the entries be one; 'iff' means 'if and only if '; $(A)^{\dagger}$ is the Moore-Penrose inverse of A.

2. Preliminaries

In this section, some basic definitions and conclusions about descriptor systems and graph theory are presented, which will be utilized in the following part.

2.1 Related results of descriptor systems

Consider a linear descriptor system described by

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where E, $A \in R^{n\times n}$, $B \in R^{n\times r}$, $C \in R^{m\times n}$. We shall assume that (E, A) is regular and $rank(E) = q \le n$.

Firstly, some definitions on linear descriptor systems are introduced [34]:

Definition 1 If there exists a scalar $s_0 \in C$ such that $\det(s_0E - A) \ne 0$, then matrix pair (E, A) is regular.

Definition 2 If $\deg(\det(sE - A)) = rankE$ holds for any $s \in C$, then matrix pair (E, A) is impulse-free.

Definition 3 If all the roots of $\det(sE - A) = 0$ have negative real parts, then matrix pair (E, A) is stable.

Definition 4 If (E, A) is impulse-free and stable, then matrix pair (E, A) is admissible.

Secondly, simple criteria for system analysis are given [34]:

Lemma 1 (E, A) is regular and impulse-free iff

$$rank\begin{bmatrix} E & 0 \\ A & E \end{bmatrix} = n + rankE \tag{2}$$

Lemma 2 (E, A, B) is stabilizable iff the matrix $\begin{bmatrix} sE - A & B \end{bmatrix}$ has full row rank for any $s \in \bar{C}^+$; (E, A, C) is detectable iff the matrix $\begin{bmatrix} sE - A \\ C \end{bmatrix}$ has full column rank for any $s \in \bar{C}^+$.

Finally, suppose (E, A) is regular and impulse-free, then there exist two nonsingular matrices U and S satisfying

$$UES = \begin{bmatrix} I_q & \\ & 0 \end{bmatrix}, \ UAS = \begin{bmatrix} A_1 & \\ & I \end{bmatrix}, \ UB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \ CS = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{3}$$

Suppose further that (E, A) is stable under the circumstance, it follows from Definition 3 that A_1 is stable as well.

2.2 Basic concepts about graph theory

A directed graph (digraph) $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ is defined to consist of a vertex set $\mathrm{V} = \{v_1, v_2, \cdots, v_N\}$ and an arc set $\mathrm{E} \subseteq \mathrm{V} \times \mathrm{V}$. For an arc $(v_i, v_j) \in \mathrm{E}$, v_i is called the parent vertex and v_j the child vertex. All the vertices, who have common child vertex v_i, comprise the neighbor set of vertex v_i, denoting as $\mathrm{N}_i = \{v_j \in \mathrm{V} : (v_j, v_i) \in \mathrm{E}\}$. For a finite nonempty sequence $\xi = v_1 e_1 v_2 \cdots v_{k-1} e_{k-1} v_k$ with $e_i = (v_i, v_{i+1}) \in \mathrm{E}$ ($i = 1, 2, \cdots, k-1$). If none of the arcs and vertices is same, then ξ is called a directed path. Furthermore, a directed path ξ, whose origin v_1 and terminus v_k coincide, is called a directed circle. For a digraph G, suppose (1) vertex v_n has no parent, (2) every other vertex is connected by a directed path starting from v_n, (3) G has no directed circles, then G is called a directed tree and v_n a root. In addition, if there exists an arc subset $\mathrm{E}' \subseteq \mathrm{E}$ such that $(\mathrm{V}, \mathrm{E}')$ is a directed tree, then G is said to contain a spanning tree.

$\mathrm{D} = (a_{ij})_{N \times N}$ represents an adjacency matrix whose elements a_{ij} $(i, j = 1, 2, \cdots, N)$ are defined by $a_{ij} > 0$ if $(v_j, v_i) \in \mathrm{E}$ and $a_{ij} = 0$ otherwise.

Besides, it is assumed that digraph G has no repeated arcs and self-loop, which means $a_{ii}=0$. The Laplacian matrix $L=(l_{ij})\in R^{N\times N}$ is defined as: $l_{ii}=\sum_{j\neq i}a_{ij}$ and $l_{ij}=-a_{ij}$ for $i\neq j$. It is obvious that $L\mathbf{1}_N=0$.

3. Problem Formulation

Consider a group of agents consisting of N followers and one leader. The dynamics of followers are general descriptor systems given by

$$\begin{cases} E\dot{x}_i(t)=Ax_i(t)+Bu_i(t) \\ y_i(t)=Cx_i(t) \end{cases}, \quad i=1,2,\cdots,N \tag{4}$$

where $x_i(t)\in R^n$ is the state, $u_i(t)\in R^r$ the control input and $y_i(t)\in R^m$ the output of ith follower. The output of the leader is $y_d(t)\in R^m$ (i.e., reference signal).

Denote

$$x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix},\quad u(t)=\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_N(t) \end{bmatrix},\quad y(t)=\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_N(t) \end{bmatrix}$$

then $x(t)\in R^{nN}$, $u(t)\in R^{rN}$, $y(t)\in R^{mN}$, and the global dynamics of the followers is written as

$$\begin{cases} (I_N\otimes E)\dot{x}(t)=(I_N\otimes A)x(t)+(I_N\otimes B)u(t) \\ y(t)=(I_N\otimes C)x(t) \end{cases} \tag{5}$$

The fundamental communication topology among $N+1$ agents can be depicted by a digraph $\bar{\mathrm{G}}$, where vertex v_0 denotes leader and $v_i|_{i\neq 0}$ the ith follower. If jth ($j=1,2,\cdots,N$) follower can sense the leader, then (v_0,v_j) exists with weight $m_j>0$ and $m_j=0$ otherwise. In $\bar{\mathrm{G}}$, $M=diag(m_1,m_2,\cdots,m_N)$ refers to leader adjacent matrix, and $H=L+M$ expresses the connectivity of $\bar{\mathrm{G}}$, where L is the Laplacian matrix corresponding to the communication topology G of followers. Using the properties of L, it can be easily shown that $h_{ii}=(l_{ii}+m_i)>0$, $h_{ij}=-a_{ij}$ for $i\neq j$, and also $H\mathbf{1}_N=M\mathbf{1}_N$.

For the convenience of studying, we first make an assumption on the digraph $\bar{\mathrm{G}}$.

A1: The digraph $\bar{\mathrm{G}}$ is acyclic and contains a directed spanning tree, taking vertex v_0 as its root.

Under assumption A1, the following lemma shows the relationship between the matrix H and digraph $\bar{\mathrm{G}}$.

Lemma 3[35] All the eigenvalues of matrix H have positive real parts iff assumption A1 holds.

Defining the regulated error between output of follower v_i and reference signal as:

$$\xi_i(t) = y_i(t) - y_d(t)\ ,\quad i = 1, 2, \cdots, N \tag{6}$$

The cooperative optimal preview tracking problem is to design distributed optimal preview controller $u_i(t)$ for each follower v_i ($i = 1, 2, \cdots, N$), such that $\lim_{t\to\infty} \xi_i(t) = 0$ ($i = 1, 2, \cdots, N$) for all admissible initial conditions $x_i(0) = x_{i0}$.

However, for each follower in communication topology $\bar{\mathrm{G}}$, $y_d(t)$ (and thereby $\xi_i(t)$) is not always measurable. Accounting for the outputs of neighbors of v_i, a virtual regulated error is defined as below

$$e_i(t) = \sum_{j\in \mathrm{N}_i} a_{ij}(y_i(t) - y_j(t)) + m_i(y_i(t) - y_d(t)) \tag{7}$$

where a_{ij} ($j \in \mathrm{N}_i$) and m_i are elements of matrices D and M, respectively.

Let us express the global virtual regulated error as

$$e(t) = \begin{bmatrix} e_1^T(t) & e_2^T(t) & \cdots & e_N^T(t) \end{bmatrix}^T$$

and using (7), we have

$$e(t) = (H \otimes I_m) y(t) - (M \otimes I_m) y_r(t) \tag{8}$$

with $y_r(t) = \mathbf{1}_N \otimes y_d(t)$. Also, defining the global regulated error as

$$\xi(t) = \begin{bmatrix} \xi_1^T(t) & \xi_2^T(t) & \cdots & \xi_N^T(t) \end{bmatrix}^T$$

and noting the equality $H\mathbf{1}_N = M\mathbf{1}_N$, (8) can be further expressed as

$$e(t) = (H \otimes I_m)\xi(t) \tag{9}$$

It follows from assumption A1 and Lemma 3 that H is nonsingular, which, together with (9), implies that $\lim_{t\to\infty} e(t) = 0$ iff $\lim_{t\to\infty} \xi(t) = 0$. In order to achieve cooperative optimal preview tracking, we present the following quadratic performance function for systems (4)

$$J = \sum_{i=1}^{N} \int_0^{\infty} (e_i^T(t)Q_{ei}e_i(t) + \dot{x}_i^T(t)Q_{xi}\dot{x}_i(t) + \dot{u}_i^T(t)R_i\dot{u}_i(t))dt \tag{10}$$

where $Q_{ei} \in R^{m\times m}$ and $R_i \in R^{r\times r}$ ($i = 1, 2, \cdots, N$) are positive definite matrices, $Q_{xi} \in R^{n\times n}$ is positive semi-definite matrix with the following form

$$Q_{xi} = S^{-T}\begin{bmatrix} Q_i & 0 \\ 0 & 0 \end{bmatrix} S^{-1}, \quad i = 1, 2, \cdots, N \tag{11}$$

in which $Q_i \in R^{q\times q}$ is positive definite matrix and S is given in (3).

Noting that $\dot{u}_i(t)$ is used in (10) rather than $u_i(t)$, integration of $e_i(t)$ will be contained in distributed controllers [25, 26], which has the ability to eliminate the static output errors possibly generated in the tracking process.

Before approaching the problem, we still need some standard assumptions.

A2: Reference signal $y_d(t)$ is a piecewise differentiable function with finite discontinuity points in $[0, +\infty)$, satisfying

$$\lim_{t\to\infty} y_d(t) = \bar{y}_d, \quad \lim_{t\to\infty} \dot{y}_d(t) = 0$$

where $\bar{y}_d$ is a constant vector. Moreover, $y_d(t)$ is previewable, i.e., the values of $y_d(\tau)$ are available for systems (4) in $\{\tau \mid t \le \tau \le t + l_r\}$ at each instant t, where l_r is called the preview length.

A3: (E, A) is impulse-free.

A4: (E, A, B) is stabilizable and $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ has full row rank.

A5: (E, A, C) is detectable.

Remark 1 Assumption A2 implies that $y_d(t)$ and $\dot{y}_d(t)$ are bounded in $[0, +\infty)$. Assumption A3 ensures that (3) holds, which helps to reduce the optimal regulation problems of descriptor subsystems to those of reduced-order normal ones. Assumptions A1, A4 and A5 all together guarantee the existence of distributed optimal preview controllers.

We conclude this section by providing the following lemma, whose proof is postponed until appendix A.

Lemma 4 Consider the following descriptor system

$$E\dot{x}(t) = Ax(t) + \delta(t), \quad x(t_0) = x_0 \in R^n \tag{12}$$

suppose that

i (E, A) is admissible,

ii $\delta(t) \in R^n$ is bounded in $[0, +\infty)$ and satisfies $\lim_{t\to\infty} \delta(t) = 0$,

then system (12) is asymptotically stable.

4. Design of Distributed optimal preview controllers

4.1 Construction of Augmented system

In what follows, the approach of preview control theory is utilized to construct an augmented system, containing global virtual regulated error $e(t)$ and the derivation of global state vector $x(t)$.

Differentiating both sides of (5) and (8) with respect to t gives

$$\dot{e}(t) = (H \otimes C)\dot{x}(t) - (M \otimes I_m)\dot{y}_r(t) \tag{13}$$

$$(I_N \otimes E)\ddot{x}(t) = (I_N \otimes A)\dot{x}(t) + (I_N \otimes B)\dot{u}(t) \tag{14}$$

Introducing a new augmented state

$$\bar{z}(t)=\begin{bmatrix} e(t) \\ \dot{x}(t) \end{bmatrix}$$

one obtains

$$\bar{E}\dot{\bar{z}}(t)=\bar{A}\bar{z}(t)+\bar{B}\dot{u}(t)-\bar{D}\dot{y}_r(t) \tag{15a}$$

with

$$\bar{E}=\begin{bmatrix} I & 0 \\ 0 & I_N\otimes E \end{bmatrix},\ \bar{A}=\begin{bmatrix} 0 & H\otimes C \\ 0 & I_N\otimes A \end{bmatrix},\ \bar{B}=\begin{bmatrix} 0 \\ I_N\otimes B \end{bmatrix},\ \bar{D}=\begin{bmatrix} M\otimes I_N \\ 0 \end{bmatrix}$$

According to the control purpose, the observation equation is taken as

$$e(t)=\bar{C}\bar{z}(t),\quad \bar{C}=\begin{bmatrix} I & 0 \end{bmatrix} \tag{15b}$$

then (15) is the augmented system.

In terms of the variables in (15), the quadratic performance function (10) can be expressed as follows

$$J=\int_0^{\infty}(\bar{z}^T(t)Q_z\bar{z}(t)+\dot{u}^T(t)R\dot{u}(t))dt \tag{16}$$

with $Q_z=diag(Q_e,Q_x)$, $Q_e=diag(Q_{e1},Q_{e2},\cdots,Q_{eN})$, $Q_x=diag(Q_{x1},Q_{x2},\cdots,Q_{xN})$, $R=diag(R_1,R_2,\cdots,R_N)$.

4.2 Problem Conversion

In subsection 4.1, by establishing an augmented system, the cooperative tracking problem for descriptor multi-agent systems has been transformed into a global optimal regulation problem for system (15) relating to performance function (16). However, when it comes to the cooperative optimal preview tracking problem for large-scale multi-agent systems, computational complexity of controller $\dot{u}(t)$ will increase dramatically. In consideration of acyclic assumption in digraph $\bar{\mathrm{G}}$, a proper transformation will be adopted to convert system (15) to a decoupling form, which can be exploited to design distributed optimal preview controllers to implement the cooperative preview tracking.

Based on assumption A1, we rename the vertices in G such that $i>j$ if $(v_j,v_i)\in\mathrm{E}$. Hence, H becomes a lower triangular matrix correspondingly. Selecting

$T=\left[T_1;T_2;\cdots;T_N\right]$, where matrix T_k is defined as

$$T_k=\begin{bmatrix} i_{(k-1)m+1} \\ \vdots \\ i_{km} \\ i_{Nm+(k-1)n+1} \\ \vdots \\ i_{Nm+kn} \end{bmatrix}$$

in which i_r is rth row of identity matrix $I_{N(m+n)}$. Applying the coordinate transformation $\bar{z}=T^{-1}\tilde{z}$ to (15a) yields

$$\begin{cases} \tilde{E}\dot{\tilde{z}}(t)=\tilde{A}\tilde{z}(t)+\tilde{B}\dot{u}(t)-\tilde{D}\dot{y}_r(t) \\ e(t)=\tilde{C}\tilde{z}(t) \end{cases} \tag{17}$$

where

$$\tilde{z}=\left[\tilde{z}_1^T,\tilde{z}_2^T,\cdots,\tilde{z}_N^T\right]^T,\quad \tilde{z}_k=\begin{bmatrix} e_k \\ \dot{x}_k \end{bmatrix}$$

$$\tilde{E}=T\bar{E}T^{-1}=diag(\tilde{E}_1,\tilde{E}_2,\cdots,\tilde{E}_N),\quad \tilde{E}_k=\begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix}$$

$$\tilde{B}=T\bar{B}=diag(\tilde{B}_1,\tilde{B}_2,\cdots,\tilde{B}_N),\quad \tilde{B}_k=\begin{bmatrix} 0 \\ B \end{bmatrix}$$

$$\tilde{C}=\bar{C}T^{-1}=diag(\tilde{C}_1,\tilde{C}_2,\cdots,\tilde{C}_N),\quad \tilde{C}_k=\begin{bmatrix} I & 0 \end{bmatrix}$$

$$\tilde{D}=T\bar{D}=diag(\tilde{D}_1,\tilde{D}_2,\cdots,\tilde{D}_N),\quad \tilde{D}_k=\begin{bmatrix} m_k I \\ 0 \end{bmatrix}$$

$$\tilde{A}=T\bar{A}T^{-1}=\begin{bmatrix} \tilde{A}_1 & & & \\ A_{21} & \ddots & & \\ \vdots & \ddots & \tilde{A}_{N-1} & \\ A_{N1} & \cdots & A_{N(N-1)} & \tilde{A}_N \end{bmatrix},\quad \tilde{A}_k=\begin{bmatrix} 0 & h_{kk}C \\ 0 & A \end{bmatrix},\quad A_{ij}=\begin{bmatrix} 0 & h_{ij}C \\ 0 & 0 \end{bmatrix}$$

$$k=1,2,\cdots,N,\quad i=2,3,\cdots,N,\quad i>j\geq 1$$

On the basis of the above transformation, we immediately obtain the following two lemmas.

Lemma 5 Under assumption A1, $(\bar{E},\bar{A},\bar{B})$ is stabilizable iff $(\tilde{E}_k,\tilde{A}_k,\tilde{B}_k)$

($k=1,2,\cdots,N$) is stabilizable.

Proof. See Appendix B.

Lemma 6 Under assumption A1, $(\bar{E},\bar{A})$ is impulse-free iff $(\tilde{E}_k,\tilde{A}_k)$ ($k=1,2,\cdots,N$) is impulse-free.

Proof. Based on Lemma 1, the proof of this result is quite similar to that of Lemma 5 and so is omitted.

Remark 2 With Lemma 1 and Lemma 2, it is easy to prove that under assumption A1(guaranteeing $h_{kk}>0$), the necessary and sufficient conditions for the impulse-freeness of $(\tilde{E}_k,\tilde{A}_k)$ and the stabilizability of $(\tilde{E}_k,\tilde{A}_k,\tilde{B}_k)$ are assumptions A3 and A4, respectively. Hence, the immediate consequences of Lemma 5 and Lemma 6 are that

Corollary 1 Under assumption A1, system (17) is impulse-free and stabilizable iff assumptions A3 and A4 hold, respectively.

Lemma 5 and Lemma 6 ensure the following fact, namely, if there exists a controller $\dot{u}_k(t)=\tilde{K}_k\tilde{z}_k(t)$ such that $(\tilde{E}_k,\tilde{A}_k+\tilde{B}_k\tilde{K}_k)$ is admissible for all $k=1,2,\cdots,N$, then similar to the proof of Lemma 5, it is easy to see that $(\tilde{E},\tilde{A}+\tilde{B}\tilde{K})$ is also admissible by $\dot{u}(t)=[\dot{u}_1^T(t),\dot{u}_2^T(t),\cdots,\dot{u}_N^T(t)]^T$, where $\tilde{K}$ is a diagonal matrix with diagonal element $\tilde{K}_k$. From the above consideration, we make virtual descriptor subsystem for each follower v_i as follows

$$\tilde{E}_i\dot{z}_{vi}(t)=\tilde{A}_i z_{vi}(t)+\tilde{B}_i\dot{u}_{vi}(t)-\tilde{D}_i\dot{y}_d(t),\quad i=1,2,\cdots,N \tag{18}$$

Where $z_{vi}=\begin{bmatrix}e_{vi}^T & \dot{x}_{vi}^T\end{bmatrix}^T$. Denoting $z_v=\begin{bmatrix}z_{v1}^T,z_{v2}^T,\cdots,z_{vN}^T\end{bmatrix}^T$ and expressing (18) into a compact form, which has the same dynamical characteristics as system (17). After performing the corresponding coordinate transformation to the performance function (16), we set following performance function for each virtual subsystem

$$J_i=\int_0^\infty (z_{vi}^T(t)Q_{zi}z_{vi}(t)+\dot{u}_{vi}^T(t)R_i\dot{u}_{vi}(t))dt,\quad i=1,2,\cdots,N \tag{19}$$

with $Q_{zi} = diag(Q_{ei}, Q_{xi})$.

Thus for, the original cooperative preview tracking problem has been transformed into optimal regulation problems under systems (18), for which the optimal controller $\dot{u}_{vi}(t)$ is determined to minimize the performance function (19). Once $\dot{u}_{vi}(t) = K_i z_{vi}(t) + f_i(t)$ is devised using the method of optimal control, such that the closed-loop system of (18) is asymptotically stable, $\dot{u}_i(t) = K_i \tilde{z}_i(t) + f_i(t)$ ($i = 1, 2, \cdots, N$) can be applied to system (17) to realize the initial control purpose.

Remark 3 This new framework provides a kind of distributed design approach for addressing cooperative preview tracking problem.

For system (18), suppose that assumption A3 holds, then according to (3), there exist two nonsingular matrices

$$\tilde{U} = \begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix}, \quad \tilde{S} = \begin{bmatrix} I & 0 \\ 0 & S \end{bmatrix}$$

such that

$$\tilde{U}\tilde{E}_i\tilde{S} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{U}\tilde{A}_i\tilde{S} = \begin{bmatrix} 0 & h_{ii}C_1 & h_{ii}C_2 \\ 0 & A_1 & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \tilde{U}\tilde{B}_i = \begin{bmatrix} 0 \\ B_1 \\ B_2 \end{bmatrix}, \quad \tilde{U}\tilde{D}_i = \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

Denote

$$\tilde{S}^{-1} z_{vi} = \begin{bmatrix} e_{vi} \\ \dot{x}_{vi_1} \\ \dot{x}_{vi_2} \end{bmatrix} \tag{21}$$

then system (18) is r.s.e. to

$$\begin{cases} \dot{e}_{vi}(t) = h_{ii}C_1\dot{x}_{vi_1}(t) + h_{ii}C_2\dot{x}_{vi_2}(t) - \dot{y}_d(t) \\ \ddot{x}_{vi_1}(t) = A_1\dot{x}_{vi_1}(t) + B_1\dot{u}_{vi}(t) \\ 0 = \dot{x}_{vi_2}(t) + B_2\dot{u}_{vi}(t) \end{cases}, \quad i = 1, 2, \cdots, N \tag{22}$$

Let $\tilde{z}_{vi} = \begin{bmatrix} e_{vi}^T & \dot{x}_{vi_1}^T \end{bmatrix}^T$, so it follows from $\dot{x}_{vi_2}(t) = -B_2\dot{u}_{vi}(t)$ that

$$\dot{\tilde{z}}_{vi} = \tilde{A}_{i_1}\tilde{z}_{vi} + \tilde{B}_{i_1}\dot{u}_{vi} - \tilde{D}_{i_1}\dot{y}_d(t) \tag{23}$$

where

$$\tilde{A}_{i_1} = \begin{bmatrix} 0 & h_{ii}C_1 \\ 0 & A_1 \end{bmatrix},\ \tilde{B}_{i_1} = \begin{bmatrix} -h_{ii}C_2B_2 \\ B_1 \end{bmatrix},\ \tilde{D}_{i_1} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Noting the characteristic of Q_{xi} in (11) and the coordinate transformation (21), the performance function (19) is changed into

$$\tilde{J}_i = \int_0^\infty (\tilde{z}_{vi}^T(t)\tilde{Q}_i\tilde{z}_{vi}(t) + \dot{u}_{vi}^T(t)R_i\dot{u}_{vi}(t))dt,\ i = 1, 2, \cdots, N \tag{24}$$

where $\tilde{Q}_i = diag(Q_{ei}, Q_i)$ is a positive definite matrix.

Eventually, the cooperative preview tracking problem for descriptor multi-agent systems is converted to optimal regulation problem for reduced-order subsystem (23) regarding performance function (24).

Remark 4 As compared with the results in [34], it is the special selection of Q_{xi} that makes the current paper avoid the discussion about the positive definiteness of weighted matrices in performance function after r.s.e., thereby reducing the design complexity of the controller, and meanwhile, that will also assist to establish the conclusions of the optimal preview control based on virtual descriptor subsystem.

4.3 Design of Distributed Controllers

Because $\tilde{Q}_i$ is a positive definite matrix, to ensure the existence of optimal preview controllers $\dot{u}_{vi}(t)$ ($i = 1, 2, \cdots, N$) of systems (23), it is only necessary to validate the stabilizability of $(\tilde{A}_{i_1}, \tilde{B}_{i_1})$.

Lemma 7 Under assumption A1, $(\tilde{A}_{i_1}, \tilde{B}_{i_1})$ ($i = 1, 2, \cdots, N$) is stabilizable iff assumption A4 holds.

To prove the lemma, we need the following conclusion:

Lemma 8 For a given i, $i = 1, 2, \cdots, N$, $(\tilde{A}_{i_1}, \tilde{B}_{i_1})$ is stabilizable iff $(\tilde{E}_i, \tilde{A}_i, \tilde{B}_i)$ is stabilizable.

Proof. The proof of this lemma is similar to that of Lemma 2 in [32] and so is omitted.

Proof of Lemma 7. The lemma is now a direct consequence of Corollary 1 and Lemma 8.

Based on what we have proved, the following lemma can immediately hold by using the results in [26].

Lemma 9 Suppose that $(\tilde{A}_{i_1}, \tilde{B}_{i_1})$ is stabilizable and $\tilde{Q}_i$ is positive definite ($i = 1, 2, \cdots, N$), then the optimal controller of the system (23) that minimizes quadratic performance function (24) is given by

$$\dot{u}_{vi}(t) = -R_i^{-1}\tilde{B}_{i_1}^T P_i \tilde{z}_{vi}(t) + f_i(t), \quad i = 1, 2, \cdots, N \tag{25}$$

where

$$f_i(t) = R_i^{-1}\tilde{B}_{i_1}^T \int_0^{l_r} e^{\sigma \tilde{A}_{ci}^T} P_i \tilde{D}_{i_1} \dot{y}_d(t+\sigma) d\sigma \tag{26}$$

and P_i is a $(m+q)\times(m+q)$ positive definite matrix, satisfying the algebraic Riccati equations (ARE)

$$\tilde{A}_{i_1}^T P_i + P_i \tilde{A}_{i_1} - P_i \tilde{B}_{i_1} R_i^{-1} \tilde{B}_{i_1}^T P_i + \tilde{Q}_i = 0 \tag{27}$$

Moreover, $\tilde{A}_{ci}$ is a stable matrix defined as follows:

$$\tilde{A}_{ci} = \tilde{A}_{i_1} - \tilde{B}_{i_1} R_i^{-1} \tilde{B}_{i_1}^T P_i, \quad i = 1, 2, \cdots, N \tag{28}$$

In terms of Lemma 9, a constructive method is adopted to derive optimal preview controllers associated with system (18) and quadratic performance function (19).

Theorem 1 suppose that $(\tilde{E}_i, \tilde{A}_i)$ is impulse-free, $(\tilde{E}_i, \tilde{A}_i, \tilde{B}_i)$ is stabilizable, and also $(\tilde{E}_i, \tilde{A}_i, Q_{zi}^{1/2})$ is detectable, then the optimal controller of the system (18) that minimizes quadratic performance functions (19) is

$$\dot{u}_{vi}(t) = -R_i^{-1}\tilde{B}_i^T X_i z_{vi}(t) + g_i(t) \tag{29}$$

where

$$g_i(t) = R_i^{-1}\tilde{B}_i^T X_i \tilde{S}\tilde{M}_i \int_0^{l_r} e^{\sigma \hat{A}_i} \tilde{S}^T X_i^T \tilde{D}_i \dot{y}_d(t+\sigma) d\sigma \tag{30}$$

with $\hat{A}_i = \tilde{S}^T[(\tilde{A}_i^T - X_i^T \tilde{B}_i R_i^{-1} \tilde{B}_i^T) X_i]\tilde{S}\tilde{M}_i$, $\tilde{M}_i = M_i^{\dagger}$, $M_i = \tilde{S}^T \tilde{E}_i^T X_i \tilde{S}$, and X_i meets the generalized algebraic Riccati equation (GARE)

$$\tilde{A}_i^T X_i + X_i^T \tilde{A}_i - X_i^T \tilde{B}_i R_i^{-1} \tilde{B}_i^T X_i + Q_{zi} = 0 \tag{31a}$$

$$\tilde{E}_i^T X_i = X_i^T \tilde{E}_i \geq 0 \tag{31b}$$

Alternatively, matrix pair $(\tilde{E}_i, \tilde{A}_i - \tilde{B}_i R_i^{-1} \tilde{B}_i^T X_i)$ ($i = 1, 2, \cdots, N$) is admissible.

Proof. See Appendix C.

Theorem 1 not only offers an optimal preview controller based on system (18), but its proof also provides a sort of explicit expression for the admissible solution of GARE (31). Actually, to design such a kind of controller, the key part is to obtain an admissible solution of GARE (31). In [36], a method for solving GARE (31) is presented, but the procedure relies heavily on the Jordan decomposition of a Hamilton matrix relating to (31), and also on a negative semi-definite solution of a low dimension ARE. Compared with [36], the method proposed in the current paper is easier to operate.

Under assumption A1, it has proved that assumptions A3 and A4 are necessary and sufficient conditions for the impulse-freeness of $(\tilde{E}_i, \tilde{A}_i)$ and stabilizability of $(\tilde{E}_i, \tilde{A}_i, \tilde{B}_i)$ respectively. To guarantee the existence of admissible solution of GARE (31), it still needs to prove the detectability of $(\tilde{E}_i, \tilde{A}_i, Q_{zi}^{1/2})$. The following lemma shows its sufficient condition.

Lemma 10 Under assumption A1, $(\tilde{E}_i, \tilde{A}_i, Q_{zi}^{1/2})$ ($i = 1, 2, \cdots, N$) is detectable if assumption A5 holds.

Proof. The argument is analogous to that of lemma 5.3 in [26] and so is omitted here.

4.4 Stability of the Closed-loop System

The stability of the closed-loop system of (18) is discussed below. By substituting controller (29) into virtual subsystem (18), we obtain

$$\tilde{E}_i \dot{z}_{vi}(t) = (\tilde{A}_i - \tilde{B}_i R_i^{-1} \tilde{B}_i^T X_i) z_{vi}(t) + \theta_i(t) \tag{32}$$

where

$$\theta_i(t) = \tilde{B}_i g_i(t) - \tilde{D}_i \dot{y}_d(t), \quad i = 1, 2, \cdots, N$$

Theorem 2 Suppose that assumptions A1-A5 hold, then closed-loop system (32) is asymptotically stable.

Proof. According to Lemma 4, if $(\tilde{E}_i, \tilde{A}_i - \tilde{B}_i R_i^{-1} \tilde{B}_i^T X_i)$ ($i = 1, 2, \cdots, N$) is admissible, furthermore, $\theta_i(t)$ is bounded on $[0,+\infty)$ and satisfies $\lim_{t\to\infty}\theta_i(t)=0$, then the result follows immediately.

Firstly, assumptions A1, A3-A5 guarantee that the conditions of Theorem 1 hold, therefore $(\tilde{E}_i, \tilde{A}_i - \tilde{B}_i R_i^{-1}\tilde{B}_i^T X_i)$ is admissible.

The next thing is to prove the boundedness of $\theta_i(t)$ on $[0,+\infty)$ and $\lim_{t\to\infty}\theta_i(t)=0$. From the proof of Theorem 1, we know that $\|g_i(t)\| = \|f_i(t)\|$. Due to the stability of $\tilde{A}_{ci}$, there exist constants W and $\alpha>0$, such that $\|e^{\tilde{A}_{ci}\sigma}\| \le We^{-\alpha\sigma}$ holds for all $\sigma \ge 0$. Moreover, assumption A2 implies that there exists a constant K such that $\|\dot{y}_d(t)\| \le K$, hence

$$
\begin{aligned}
\|\theta_i(t)\| &\le \|\tilde{B}_i g_i(t)\| + \|\tilde{D}_i \dot{y}_d(t)\| \\
&\le \|\tilde{B}_i\|\|g_i(t)\| + \|\tilde{D}_i\|\|\dot{y}_d(t)\| \\
&\le \|\tilde{B}_i\|\|f_i(t)\| + K\|\tilde{D}_i\| \\
&\le \|\tilde{B}_i\|\|R_i^{-1}\|\|\tilde{B}_{i_1}^T\|\int_0^{l_r}\|e^{\sigma\tilde{A}_{ci}^T}P_i\tilde{D}_{i_1}\dot{y}_d(t+\sigma)\|d\sigma + K\|\tilde{D}_i\| \\
&\le \bar{W}\int_0^{l_r} e^{-\alpha\sigma}d\sigma + K\|\tilde{D}_i\| \\
&= \bar{W}[-\frac{1}{\alpha}(e^{-\alpha l_r}-1)] + K\|\tilde{D}_i\|
\end{aligned}
$$

where $\bar{W} = \|\tilde{B}_i\|\|R_i^{-1}\|\|\tilde{B}_{i_1}^T\|\|P_i\|\|\tilde{D}_{i_1}\|WK$, and $\bar{W}[-\frac{1}{\alpha}(e^{-\alpha l_r}-1)] + K\|\tilde{D}_i\|$ is a constant, which means that $\theta_i(t)$ is bounded on $[0,+\infty)$. In addition, by using $\lim_{t\to\infty}\dot{y}_d(t)=0$ and expression of $g_i(t)$ in (30), it is straightforward to show that $\lim_{t\to\infty}\theta_i(t)=0$ holds. This completes the proof of Theorem 2.

Furthermore, it follows from the conclusion of Theorem 2, as well as the relationship between systems (17) and (18) that controller

$$
\dot{u}(t) = [\dot{u}_1^T(t), \dot{u}_2^T(t), \cdots, \dot{u}_N^T(t)]^T \qquad (33)
$$

can stabilize the state of closed-loop system of (17) to zero equilibrium point asymptotically, where

$$\dot{u}_i(t) = -R_i^{-1}\tilde{B}_i^T X_i \tilde{z}_i(t) + g_i(t),\quad i=1,2,\cdots,N \tag{34}$$

By the relationship between systems (15) and (17), we know that the closed-loop system of (15) is also asymptotically stable by using the following controller

$$\dot{u}(t) = -R^{-1}\bar{B}^T T^T X T \bar{z}(t) + g(t)$$

where $X = diag\{X_1, X_2, \cdots, X_N\}$ and $g(t) = diag\{g_1(t), g_2(t), \cdots, g_N(t)\}$. The above result implies that $\lim_{t\to\infty} e(t) = 0$, that is to say, controller (34) enables the multi-agent system (4) realize the cooperative optimal preview tracking. After integrating (34) on $[-(l_r+\varepsilon), t)$, where ε is a suitable small positive number, we summarize what we have proved as the following theorem.

Theorem 3 Suppose

(a) A1-A5 hold;

(b) Q_{ei}, R_i and Q_i, $i=1,2,\cdots,N$, are positive definite matrices,

and let $u_i(t)=0$, $y_d(t)=0$ for $t<0$, then the distributed optimal preview controller, which solves the cooperative preview tracking problem for descriptor multi-agent systems (4), is given by

$$u_i(t) = -K_{ei}\int_0^t e_i(\sigma)d\sigma - K_{xi}x_i(t) + \bar{g}_i(t),\quad i=1,2,\cdots,N \tag{35}$$

where $\bar{g}_i(t)\in R^r$ is a preview compensation satisfying

$$\bar{g}_i(t) = R_i^{-1}\tilde{B}_i^T X_i \tilde{S}\tilde{M}_i \int_0^{l_r} e^{\sigma\hat{A}_i}\tilde{S}^T X_i^T \tilde{D}_i y_d(t+\sigma)d\sigma \tag{36}$$

and $K_{ei} = R_i^{-1}\tilde{B}_i^T X_{ei}$, $K_{xi} = R_i^{-1}\tilde{B}_i^T X_{xi}$, $X_i = [X_{ei} \quad X_{xi}]$. Alternatively, the expressions of X_i, $\hat{A}_i$ and $\tilde{M}_i$ refer to Theorem 1.

Remark 5 Noting that $\tilde{D}_i = \begin{bmatrix} m_i I \\ 0 \end{bmatrix}$ in (36), $i=1,2,\cdots,N$, then based on the definition of m_i and Theorem 2, we know that it is only necessary to add preview compensation terms for a small part of followers, then the whole followers will achieve cooperative optimal preview tracking globally by exploiting distributed controller (35).

Remark 6 In essence, the distributed controller (35) is a classical state feedback controller. However, when the full set of the states are not measurable, the theory of this paper cannot be used directly and distributed estimation through observer method will be considered for preview tracking protocol design. Moreover, there might be time delay exists in the communication of the agents, which makes the observer design more challenging. Recently, functional observers and unknown input functional observers proposed in [39-41] will be very useful for this application.

4.5 Discussion

The design method proposed in current paper has excellent scalability.

Firstly, suppose that the multi-agent systems (4) are heterogeneous, namely, the dynamic of each follower v_i is described as follows:

$$\begin{cases} E_i\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) \\ y_i(t) = C_i x_i(t) \end{cases}, \quad i = 1, 2, \cdots, N \tag{37}$$

where $x_i(t) \in R^{n_i}$ denotes the state of ith follower, we shall also assume that $rank(E_i) = q_i \le n_i$, $i = 1, 2, \ldots, N$. In terms of the cooperative preview tracking problem for systems (37), the main procedures are almost identical with that for systems (4), the major change is reflected in two aspects. one is the substitution of

$$\tilde{E}_i = \begin{bmatrix} I & 0 \\ 0 & E_i \end{bmatrix}, \; \tilde{A}_i = \begin{bmatrix} 0 & h_{ii}C_i \\ 0 & A_i \end{bmatrix}, \; \tilde{B}_i = \begin{bmatrix} 0 \\ B_i \end{bmatrix}, \; \tilde{S}_i = \begin{bmatrix} I & 0 \\ 0 & S_i \end{bmatrix}$$

for $\tilde{E}_i$, $\tilde{A}_i$, $\tilde{B}_i$ and $\tilde{S}_i$ in (18). The other is to perform r.s.e. to each descriptor virtual subsystem (18) corresponding to (37). Thus, the distributed optimal preview controller for solving cooperative preview tracking problem is given by

$$u_i(t) = -K_{ei}\int_0^t e_i(\sigma)d\sigma - K_{xi}x_i(t) + \bar{g}_i'(t), \quad i = 1, 2, \cdots, N \tag{38}$$

where

$$\bar{g}_i'(t) = R_i^{-1}\tilde{B}_i^T X_i \tilde{S}_i \tilde{M}_i \int_0^{l_r} e^{\sigma\hat{A}_i}\tilde{S}_i^T X_i^T \tilde{D}_i y_d(t+\sigma)d\sigma \tag{39}$$

Secondly, suppose that systems (4) are not impulse-free, but satisfy

A6: Systems (4) are impulse controllable and impulse observable.

Now, in order to tackle the cooperative preview tracking problem under the framework proposed in the current paper, pre-feedback is needed to transform systems (4) to impulse-free systems. It has proved in [34] that there exist output feedback controllers such that the closed-loop systems of (4) are impulse-free iff systems (4) are both impulse controllable and impulse observable. Therefore, we first need to design pre-feedback controllers for each follower in (4) as follows:

$$u_i(t) = Ky_i(t) + v_i(t), \quad i = 1, 2, \cdots, N \tag{40}$$

where $v_i(t) \in R^m$ is new auxiliary input. Substituting (40) into systems (4) gives

$$\begin{cases} E\dot{x}_i(t) = (A + BKC)x_i(t) + Bv_i(t) \\ y_i(t) = Cx_i(t) \end{cases} \tag{41}$$

Compared with Theorem 3, we present the following result

Corollary 2 Suppose

(c) A1, A2 and A4-A6 hold;

(d) Q_{ei}, R_i and Q_i, $i = 1, 2, \cdots, N$, are positive definite matrices,

and let $u_i(t) = 0$, $y_d(t) = 0$ for $t < 0$, then the distributed optimal preview controller, which solves the cooperative preview tracking problem for descriptor multi-agent systems (4), is given by

$$u_i(t) = -K_{ei}\int_0^t e_i(\sigma)d\sigma + (KC - K_{xi})x_i(t) + \bar{g}_i(t), \quad i = 1, 2, \cdots, N \tag{42}$$

where $\bar{g}_i(t) \in R^r$ is a preview compensation satisfying

$$\bar{g}_i(t) = R_i^{-1}\tilde{B}_i^T X_i \tilde{S}\tilde{M}_i \int_0^{l_r} e^{\sigma \hat{A}_i} \tilde{S}^T X_i^T \tilde{D}_i y_d(t+\sigma)d\sigma \tag{43}$$

Remark 6 According to systems (41), all the $\tilde{A}_i$ in systems (18) are needed to be replaced by $\tilde{A}_i = \begin{bmatrix} 0 & h_{ii}C \\ 0 & A + BKC \end{bmatrix}$. Moreover, due to the fact that output feedback does not change the stabilizability and detectability of systems (41), thus, under assumption A1, assumption A4 is still the necessary and sufficient condition for the stabilizability of $(\tilde{E}_i, \tilde{A}_i, \tilde{B}_i)$, and assumption A5 the sufficient condition for the detectability of $(\tilde{E}_i, \tilde{A}_i, Q_{zi}^{1/2})$.

Thirdly, if the virtual regulating error $e_i(t)$ has the following form

$$e_i(t)=\frac{1}{h_{ii}}(\sum_{j\in \mathrm{N}_i} a_{ij}(y_i(t)-y_j(t))+m_i(y_i(t)-y_d(t)),\ \ i=1,2,\cdots,N$$

then global virtual regulating error is

$$e(t)=(\bar{H}\otimes I_m)\xi(t)$$

where $\bar{H}=\bar{D}H$, $\bar{D}=diag\left\{\frac{1}{h_{11}},\frac{1}{h_{22}},\cdots,\frac{1}{h_{NN}}\right\}$. Noting that the all of the diagonal elements of $\bar{H}$ are 1, so $\tilde{E}_i$, $\tilde{A}_i$, $\tilde{B}_i$ in (18) are changed into $\tilde{E}_i=\tilde{E}=\begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix}$, $\tilde{A}_i=\tilde{A}=\begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}$, $\tilde{B}_i=\tilde{B}=\begin{bmatrix} 0 \\ B \end{bmatrix}$. Furthermore, let the weighted matrices of performance function (10) be the same, namely, $Q_{ei}=Q_e$, $R_i=R$, $Q_{xi}=Q_x$, $i=1,2,\cdots,N$, then the gain matrices K_{ei} and K_{xi} with respect to controller $u_i(t)$ would also be the same, which will decrease the calculation complexity significantly.

Finally, if E in systems (4) is nonsingular, then the problem would degenerate into the cooperative preview tracking problem for linear multi-agent systems, which has been investigated in [33]. After establishing virtual subsystems (18) for each follower, the problem will be handled by pre-multiplying $\tilde{E}_i^{-1}$ on both sides of (18) rather than performing r.s.e. to $\tilde{E}_i$ and $\tilde{A}_i$. Consequently, it avoids not only the redundant coordinate transformations, but also the constructive process needed in Theorem 1. The results about distributed optimal preview controllers in this case are immediately obtained by analogy to Lemma 9.

5. Numerical Simulation

In this section, the effectiveness of the distributed controllers will be demonstrated by a simulation example.

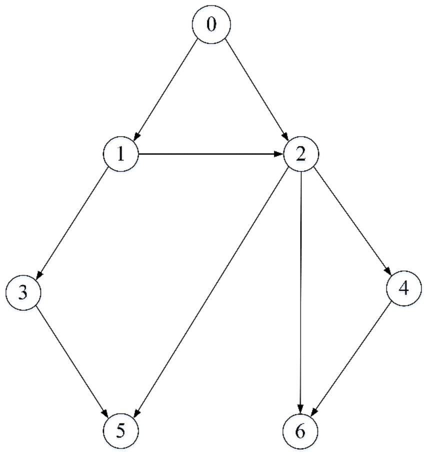

Figure 1. Communication topology of multi-agent systems

Consider a multi-agent system consisting of six followers and one leader, the dynamics of followers are (4), where

$$E=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\quad A=\begin{bmatrix}1.70 & 2.75\\ 2.85 & 4.80\end{bmatrix},\quad B=\begin{bmatrix}1\\ 2\end{bmatrix},\quad C=\begin{bmatrix}4 & 0\end{bmatrix}$$

the output of the leader is

$$y_d(t)=\begin{cases}0, & t<10\\ \frac{1}{2}(t-10), & 10\le t<16\\ 3, & t\ge 16\end{cases}$$

Based on Lemma1 and Lemma 2, it is easy to see that the E, A, B and C satisfy assumptions A3-A5. Alternatively, assuming that $y_d(t)$ is previewable.

Figure 1 shows the communication topology $\bar{\mathrm{G}}$ among followers and the leader. It is observed that the digraph $\bar{\mathrm{G}}$ contains a spanning tree and the matrix H associated with $\bar{\mathrm{G}}$ is

$$H=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ -1 & 2 & 0 & 0 & 0 & 0\\ -1 & 0 & 1 & 0 & 0 & 0\\ 0 & -1 & 0 & 1 & 0 & 0\\ 0 & -1 & -1 & 0 & 2 & 0\\ 0 & -1 & 0 & -1 & 0 & 2\end{bmatrix}$$

Because (E,A) is impulse-free, a direct computation gives rise to the matrices U and S,

$$U=\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix},\quad S=\begin{bmatrix}1 & 0 & 0\\ 1.5 & -1 & 1\\ 0 & 1 & 0\end{bmatrix}$$

In performance function (10), weighted matrices are chosen as

$$diag\{Q_{e1},Q_{e2},\cdots,Q_{e6}\}=diag\{30.0,10.0,37.5,37.5,37.5,27.5\}$$

$$diag\{Q_1,Q_2,\cdots,Q_6\}=diag\{0.01,0.01,0.01,0.01,0.01,0.01\}\otimes I_2$$

$$diag\{R_1,R_2,\cdots,R_6\}=diag\{0.275,0.225,0.225,0.225,0.225,0.225\}$$

The above selection ensures that all the conditions of Theorem 3 hold, then there exist distributed controllers such that the followers can track the leader as accurately as possible. According to Lemma 9 and Theorem 1, and using Matlab, it can be obtained that

$$diag\{K_{e1},K_{e2},\cdots,K_{e6}\}=diag\{-10.4447,-6.6667,-12.9099,-12.9099,-12.9099,-11.0554\}$$

$$K_{x1}=\begin{bmatrix}-24.4026 & 0\end{bmatrix},\quad K_{x2}=\begin{bmatrix}-27.5104 & 0\end{bmatrix}$$

$$K_{x3}=\begin{bmatrix}-27.0776 & 0\end{bmatrix},\quad K_{x4}=\begin{bmatrix}-27.0776 & 0\end{bmatrix}$$

$$K_{x5}=\begin{bmatrix}-38.0993 & 0\end{bmatrix},\quad K_{x6}=\begin{bmatrix}-35.2916 & 0\end{bmatrix}$$

To perform simulation experiment, we intend to employ the trapezoidal method to establish the iterative scheme. Specifically, substituting controllers (35) into systems (4) yields

$$E\dot{x}_i(t)=(A-BK_{xi})x_i(t)+\varphi_i(t)$$

where

$$\varphi_i(t)=-BK_{ei}\int_0^t e_i(\sigma)d\sigma+BK_{xi}x_{i0}+B\overline{g}_i(t)$$

According to Euler method and backward Euler method, the above closed-loop system can be discretized into the following two forms, namely

$$E\frac{x_i(t+T)-x_i(t)}{T}=(A-BK_{xi})x_i(t)+\overline{\varphi}_i(t)$$

and

$$E\frac{x_i(t+T)-x_i(t)}{T}=(A-BK_{xi})x_i(t+T)+\overline{\varphi}_i(t+T)$$

where T denotes iteration step and is selected as $T=0.1$ in current paper. It can be seen that T ensures the invertibility of $E-\frac{T}{2}(A-BK_{xi})$. Adding the above two formulas together and carrying on simple calculation immediately gives the following trapezoidal iterative scheme

$$x_i(t+T)=\left\{E-\frac{T}{2}(A-BK_{xi})\right\}^{-1}\left\{\left[E+\frac{T}{2}(A-BK_{xi})\right]x_i(t)+\frac{T}{2}(\bar{\varphi}_i(t)+\bar{\varphi}_i(t+T))\right\}$$

It needs to point out that the integrals in $\varphi_i(t)$ will be dealt with by the original definition of the integral.

Select the initial states of six followers as

$$x_1(0)=\begin{bmatrix}-0.03\\-0.03\end{bmatrix},\quad x_2(0)=\begin{bmatrix}-0.07\\-0.09\end{bmatrix},\quad x_3(0)=\begin{bmatrix}-0.13\\-0.15\end{bmatrix}$$

$$x_4(0)=\begin{bmatrix}0.07\\0.04\end{bmatrix},\quad x_5(0)=\begin{bmatrix}0.10\\0.17\end{bmatrix},\quad x_6(0)=\begin{bmatrix}0.03\\0.06\end{bmatrix}$$

Figure 2 to 4 show the output trajectories of the multi-agent systems (4) with the distributed controllers (35) for different preview lengths, i.e., $l_r=0.0(s)$, $l_r=0.1(s)$, $l_r=0.2(s)$.

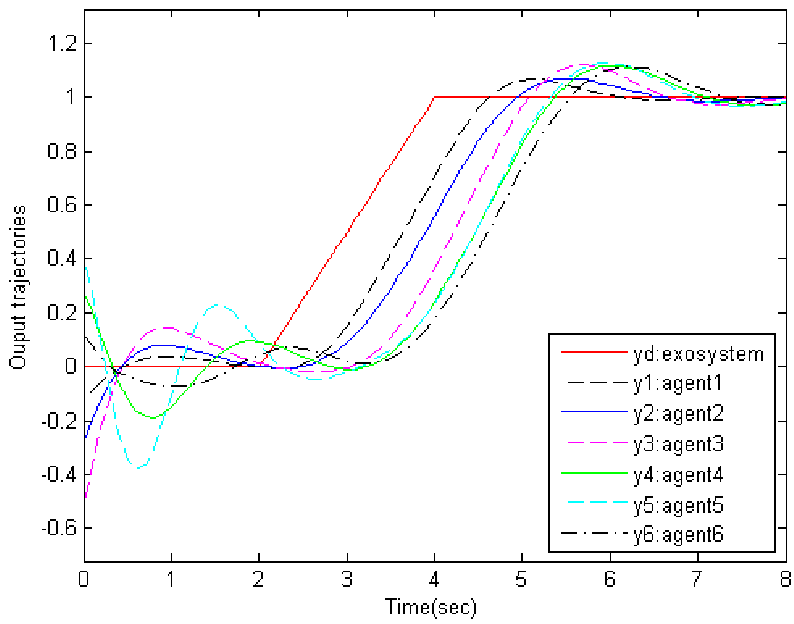

Figure 2. Output trajectories of multi-agent systems (4) for $l_r=0.0(s)$

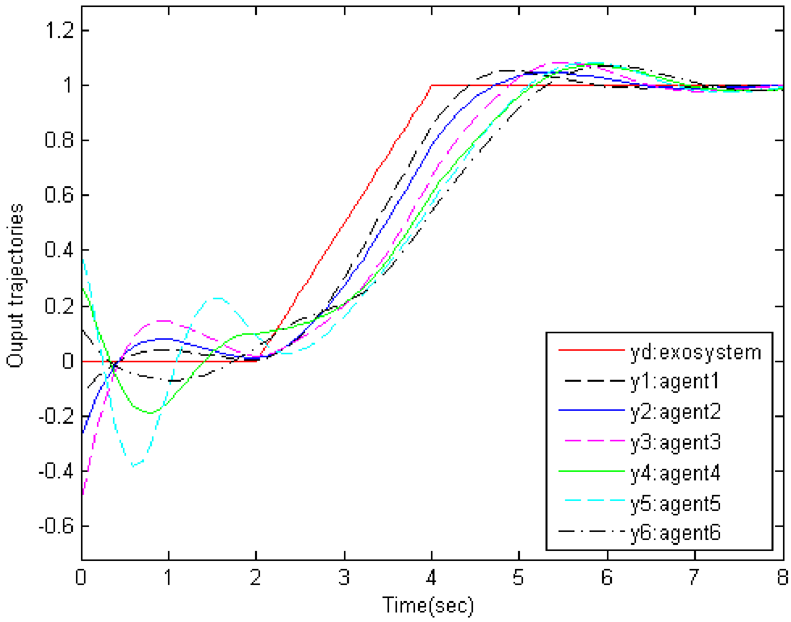

Figure 3. Output trajectories of multi-agent systems (4) for $l_r = 0.1(s)$

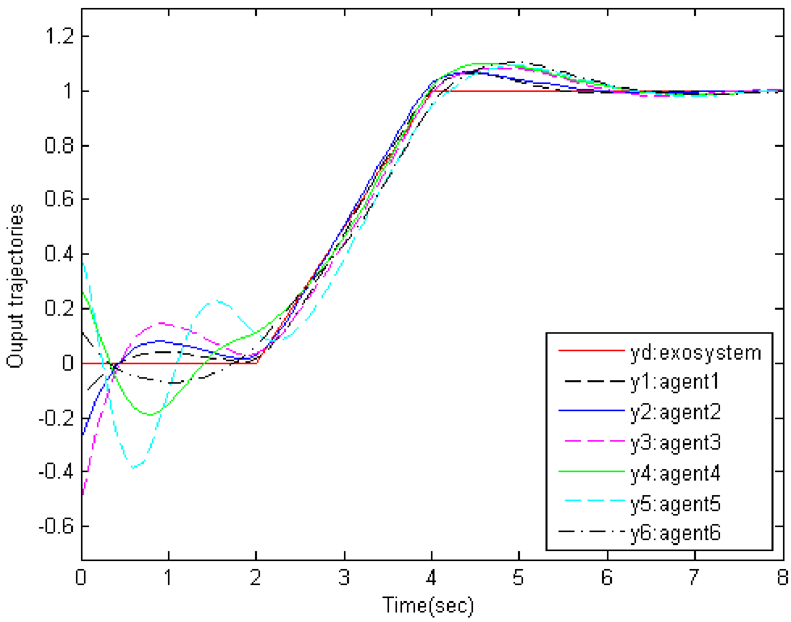

Figure 4. Output trajectories of multi-agent systems (4) for $l_r = 0.2(s)$

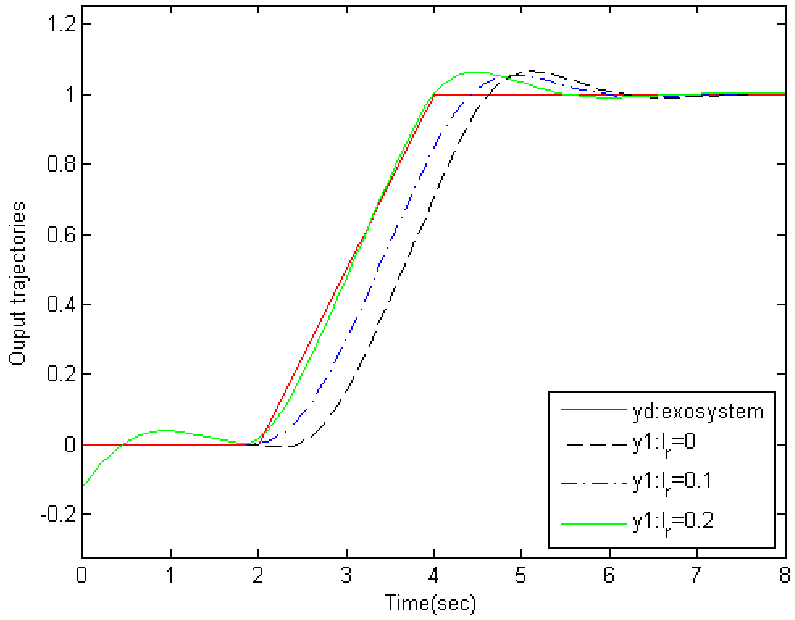

Figure 5. Output trajectories of the first agent with (35) for different preview lengths

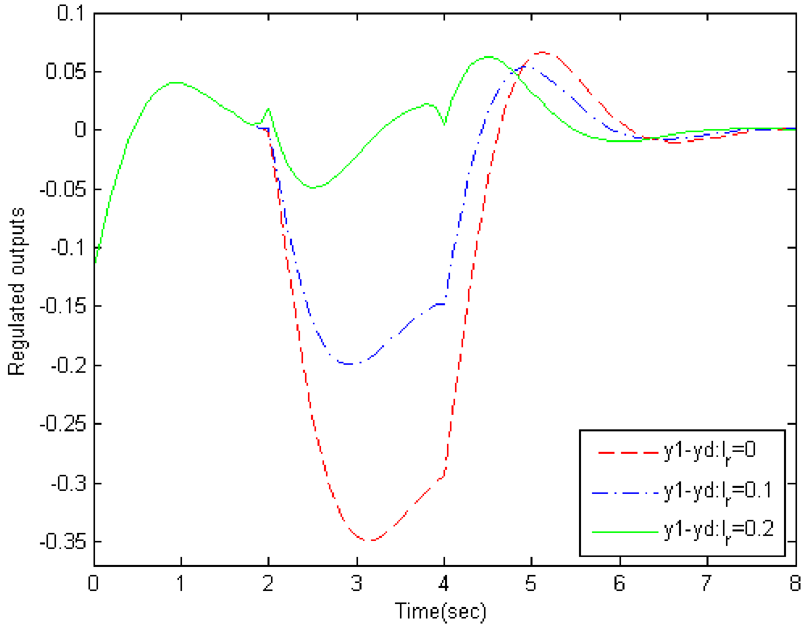

Figure 6. Regulated outputs of the first agent under different preview lengths

It can be seen that outputs of followers will converge to that of the leader no matter whether there exists preview compensation or not, which demonstrates the effectiveness of the designed distributed optimal preview controllers. Moreover, compared with figures 2-4, we find that the followers can achieve cooperative preview tracking faster and more accurately with appropriate increase of the preview length.

Figure 5 indicates the output response of the first follower for different preview lengths. It can be observed that the adjusting time is reduced significantly and the stability is reached faster for the system under the distributed optimal preview controllers. Figure 6 shows the tracking error of the first follower, which confirms the analysis above.

6. Conclusion

This paper has analyzed the cooperative preview tracking problem for impulse-free descriptor multi-agent systems, which contains a directed spanning tree without cycle. It has shown that the cooperative preview tracking problem are reached via distributed control, and the distributed optimal preview controllers have been obtained constructively based on the matrices of original systems rather than those of the reduced-order ones. Furthermore, by investigating the asymptotic stability of closed-loop descriptor subsystems, the sufficient conditions have also been presented for cooperative tracking consensus. It is worthy to mention that the distributed design framework can be extended to solve cooperative preview tracking problem for heterogeneous descriptor multi-agent systems, and also the dynamics of followers satisfying impulse controllable and impulse observable. Simulation results have indicated the superiority of the developed controllers.

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Appendices

We prove Lemma 4 by first introducing the following conclusion.

Lemma 11 [37] Consider a continuous-time linear system

$$\begin{cases} \dot{x}(t) = Ax(t) + \delta(t) \\ x(t_0) = x_0 \in R^n \end{cases}$$

if

i A is stable,

ii $\delta(t) \in R^n$ is bounded in $[0, +\infty)$ and satisfies $\lim_{t \to \infty} \delta(t) = 0$,

then the system is asymptotically stable.

A. Proof of Lemma 4

Proof. If (E, A) is admissible, then according to formula (3), there exist two nonsingular matrices U and S such that

$$UES = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad UAS = \begin{bmatrix} A_1 & 0 \\ 0 & I \end{bmatrix}$$

where A_1 is a stable matrix. By letting $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = S^{-1}x$, system (12) is r.s.e. to the following systems

$$\dot{x}_1 = A_1 x_1 + U_1 \delta, \quad x_{10} = \begin{bmatrix} I & 0 \end{bmatrix} S^{-1} x_0 \tag{A.1}$$

$$x_2 = -U_2 \delta, \quad x_{20} = \begin{bmatrix} 0 & I \end{bmatrix} S^{-1} x_0 = -U_2 \delta(0) \tag{A.2}$$

with $U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$, $U_1 \in R^{q \times n}$, $U_2 \in R^{(n-q) \times n}$.

Since system (A.1) meets all the conditions needed in Lemma 11, it follows that system (A.1) is asymptotically stable. Alternatively, from formula (A.2), we have

$$\|x_2\| = \|-U_2 \delta\| \le \|U_2\| \|\delta\|$$

It follows from condition ii that x_2 stabilizes to 0 asymptotically.

Based on the asymptotic property of x_1 and x_2, and

$$\|x\| = \left\| S \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\| \le \|S\| \left\| \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\| \le \|S\| \left(\|x_1\| + \|x_2\| \right)$$

It is evident to see that the lemma holds.

B. Proof of Lemma 5

Proof. Since nonsingular linear transformation does not change the stabilizability of linear descriptor system, it follows that $(\bar{E}, \bar{A}, \bar{B})$ and $(\tilde{E}, \tilde{A}, \tilde{B})$ have common stabilizability. Thereupon, it suffices to prove the following equivalent proposition, i.e., under assumption A1, $(\tilde{E},\tilde{A},\tilde{B})$ is stabilizable iff $(\tilde{E}_k,\tilde{A}_k,\tilde{B}_k)$ ($k=1,2,\cdots,N$) is stabilizable. According to the conclusion of Lemma 2, the proposition can be further expressed as: under assumption A1, $\begin{bmatrix} s\tilde{E}-\tilde{A} & \tilde{B} \end{bmatrix}$ is of full row rank iff $\begin{bmatrix} s\tilde{E}_k-\tilde{A}_k & \tilde{B}_k \end{bmatrix}$($k=1,2,\cdots,N$) is of full row rank for all $s\in\bar{C}^{+}$. The property that elementary transformation does not change the rank of a matrix will be used repeatedly in the following proofs. Noting that

$$\begin{bmatrix} s\tilde{E}-\tilde{A} & \tilde{B} \end{bmatrix}=\left[\begin{array}{cc:cc:c:cc:c:c:c:c} sI & -h_{11}C & & & & & & 0 & & & \\ 0 & sE-A & & & & & & B & & & \\ \hdashline 0 & -h_{21}C & sI & -h_{22}C & & & & & 0 & & \\ 0 & 0 & 0 & sE-A & & & & & B & & \\ \hdashline \vdots & \vdots & \vdots & \vdots & \ddots & & & & & \ddots & \\ \hdashline 0 & -h_{N1}C & 0 & -h_{N2}C & \cdots & sI & -h_{NN}C & & & & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & sE-A & & & & B \end{array}\right]$$

Performing simple column exchange for the above formula gives

$$\left[\begin{array}{ccc:ccc:c:ccc} sI & -h_{11}C & 0 & & & & & & & \\ 0 & sE-A & B & & & & & & & \\ \hdashline 0 & -h_{21}C & 0 & sI & -h_{22}C & 0 & & & & \\ 0 & 0 & 0 & 0 & sE-A & B & & & & \\ \hdashline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & & & \\ \hdashline 0 & -h_{N1}C & 0 & 0 & -h_{N2}C & 0 & \cdots & sI & -h_{NN}C & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & sE-A & B \end{array}\right] \overset{\text{def}}{=} V(s)$$

The rank of $V(s)$ will be discussed in two cases, i.e., $s=0$, as well as $\mathrm{Re}(s)\geq 0$ and $s\neq 0$.

For $s=0$, using $h_{ii}>0$ (invoking assumption A1), the block matrix $-h_{ji}C$ ($j>i\geq 1$) that has the same column pivot element $-h_{ii}C$ will be eliminated. That is

$$\left[\begin{array}{ccc:ccc:c:ccc} 0 & -h_{11}C & 0 & & & & & & & \\ 0 & -A & B & & & & & & & \\ \hdashline & & & 0 & -h_{22}C & 0 & & & & \\ & & & 0 & -A & B & & & & \\ \hdashline & & & & & & \ddots & & & \\ \hdashline & & & & & & & 0 & -h_{NN}C & 0 \\ & & & & & & & 0 & -A & B \end{array}\right]$$

It is evident that $V(0)$ has full row rank iff

$$rank\begin{bmatrix} h_{kk}C & 0 \\ A & B \end{bmatrix} = m+n, \text{ (full row rank) } k=1,2,\cdots,N \quad \text{(B.1)}$$

For $\mathrm{Re}(s) \geq 0$ and $s \neq 0$, the matrix $-h_{ij}C\,(i > j \geq 1)$ that is on the same row with sI (nonsingular) can be eliminated. Then the result is

$$\left[\begin{array}{ccc:ccc:c:ccc} sI & -h_{11}C & 0 & & & & & & & \\ 0 & sE-A & B & & & & & & & \\ \hdashline & & & sI & -h_{22}C & 0 & & & & \\ & & & 0 & sE-A & B & & & & \\ \hdashline & & & & & & \ddots & & & \\ \hdashline & & & & & & & sI & -h_{NN}C & 0 \\ & & & & & & & 0 & sE-A & B \end{array}\right]$$

It can be observed that $V(s)$ has full row rank for $\mathrm{Re}(s) \geq 0$ and $s \neq 0$ iff

$$rank\begin{bmatrix} sI & -h_{kk}C & 0 \\ 0 & sE-A & B \end{bmatrix} = m+n, \text{ (full row rank) } k=1,2,\cdots,N \quad \text{(B.2)}$$

According to Lemma 2, (B.1) together with (B.2) constitutes the necessary and sufficient condition for the stabilizability of $(\tilde{E}_k, \tilde{A}_k, \tilde{B}_k)$ ($k=1,2,\cdots,N$). This completes the proof of Lemma 5.

C. Proof of Theorem 1

Proof. Since the conditions of Theorem 1 can ensure that Lemma 9 holds, it follows that ARE in Lemma 9 has a unique positive definite solution matrix P_i. In consistent with the partition $\tilde{A}_{i_1}$, partition P_i as

$$P_i = \begin{bmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{i_{12}}^T & P_{i_{22}} \end{bmatrix}, \quad i = 1, 2, \cdots, N$$

then (27) can be expressed as

$$\begin{bmatrix} 0 & 0 \\ h_{ii}C_1^T & A_1^T \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{i_{12}}^T & P_{i_{22}} \end{bmatrix} + \begin{bmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{i_{12}}^T & P_{i_{22}} \end{bmatrix} \begin{bmatrix} 0 & h_{ii}C_1 \\ 0 & A_1 \end{bmatrix} - \begin{bmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{i_{12}}^T & P_{i_{22}} \end{bmatrix}$$

$$\begin{bmatrix} -h_{ii}C_2B_2 \\ B_1 \end{bmatrix} R_i^{-1} \begin{bmatrix} -h_{ii}B_2^TC_2^T & B_1^T \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{i_{12}}^T & P_{i_{22}} \end{bmatrix} + \begin{bmatrix} Q_{ei} & 0 \\ 0 & Q_i \end{bmatrix} = 0 \qquad \text{(C.1)}$$

The trick of the proof is to expand (C.1) into the following form equivalently

$$\begin{bmatrix} 0 & 0 & 0 \\ h_{ii}C_1^T & A_1^T & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & h_{ii}C_1 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & I \end{bmatrix} - \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -h_{ii}C_2B_2 \\ B_1 \\ B_2 \end{bmatrix} R_i^{-1} \begin{bmatrix} -h_{ii}B_2^TC_2^T & B_1^T & B_2^T \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} Q_{ei} & 0 & 0 \\ 0 & Q_i & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \qquad \text{(C.2)}$$

and the key equalities are

$$\begin{bmatrix} I & 0 & -h_{ii}C_2 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} 0 & h_{ii}C_1 & h_{ii}C_2 \\ 0 & A_1 & 0 \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} 0 & h_{ii}C_1 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & I \end{bmatrix} \qquad \text{(C.3a)}$$

$$\begin{bmatrix} I & 0 & -h_{ii}C_2 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} 0 \\ B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} -h_{ii}C_2B_2 \\ B_1 \\ B_2 \end{bmatrix} \qquad \text{(C.3b)}$$

Substituting (C.3) into (C.2) gives

$$\begin{bmatrix} 0 & 0 & 0 \\ h_{ii}C_1^T & A_1^T & 0 \\ h_{ii}C_2^T & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -h_{ii}C_2^T & 0 & I \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I & 0 & -h_{ii}C_2 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} 0 & h_{ii}C_1 & h_{ii}C_2 \\ 0 & A_1 & 0 \\ 0 & 0 & I \end{bmatrix} - \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & -h_{ii}C_2 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} 0 \\ B_1 \\ B_2 \end{bmatrix} R_i^{-1}$$

$$\begin{bmatrix} 0 & B_1^T & B_2^T \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -h_{ii}C_2^T & 0 & I \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} Q_{ei} & 0 & 0 \\ 0 & Q_i & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \qquad \text{(C.4)}$$

Pre-multiplying (C.4) by $\tilde{S}^{-T}$ and post-multiplying by its transposition, and then putting

$$X_i = \tilde{U}^T \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -h_{ii}C_2^T & 0 & I \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tilde{S}^{-1} \tag{C.5}$$

Consequently, a formula with respect to X_i is obtained based on the restricted equivalent forms of $\tilde{A}_i$ and $\tilde{B}_i$ in (20), namely

$$\tilde{A}_i^T X_i + X_i^T \tilde{A}_i - X_i^T \tilde{B}_i R_i^{-1} \tilde{B}_i^T X_i + Q_{zi} = 0$$

Furthermore, noting

$$\begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & -h_{ii}C_2 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -h_{ii}C_2^T & 0 & I \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{C.6}$$

and repeating the preceding operation on it gives

$$\tilde{S}^{-T} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & -h_{ii}C_2 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tilde{U}\tilde{U}^{-1} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} \tilde{S}^{-1} =$$

$$\tilde{S}^{-T} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} \tilde{U}^{-T}\tilde{U}^T \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -h_{ii}C_2^T & 0 & I \end{bmatrix} \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tilde{S}^{-1} \tag{C.7}$$

then according to the restricted equivalent form of $\tilde{E}_i$ in (20), (C.7) can be written as

$$\tilde{E}_i^T X_i = X_i^T \tilde{E}_i$$

the conclusion about $\tilde{E}_i^T X_i \geq 0$ follows from (C.6), and the proof of admissibility of $(\tilde{E}_i, \tilde{A}_i - \tilde{B}_i R_i^{-1} \tilde{B}_i^T X_i)$ can refer to Corollary 5.3.1 in [36].

Next, we get the expression of $g_i(t)$ from $f_i(t)$ in Lemma 9. Before expanding $f_i(t)$, $i = 1, 2, \cdots, N$, we first obtain a key formula as follows based on (C.5)

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ h_{ii}C_2^T & 0 & I \end{bmatrix} \tilde{U}^{-T} X_i \tilde{S} = \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{C.6}$$

Pre-multiplying both sides of (C.6) by $\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$ yields

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} \tilde{U}^{-T} X_i \tilde{S} = \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{C.7}$$

Applying restricted equivalent form of $\tilde{E}_i$ to (C.7) gives

$$\tilde{S}^T \tilde{E}_i^T X_i \tilde{S} = \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{C.8}$$

Denoting $M_i = \tilde{S}^T \tilde{E}_i^T X_i \tilde{S}$, then

$$M_i = \begin{bmatrix} P_{i_{11}} & P_{i_{12}} & 0 \\ P_{i_{12}}^T & P_{i_{22}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{C.9}$$

We now turn to the expansion of $f_i(t)$. As P_i is a positive definite matrix, so (26) is equivalent to

$$f_i(t) = R_i^{-1} \tilde{B}_{i_1}^T P_i P_i^{-1} \int_0^{l_r} \exp(\sigma \tilde{A}_{ci}^T P_i P_i^{-1}) P_i \tilde{D}_{i_1} \dot{y}_d(t+\sigma) d\sigma \tag{C.10}$$

With the derivation process illustrated by (C.1)-(C.7) and the expression of (C.9), $R_i^{-1} \tilde{B}_{i_1}^T P_i P_i^{-1}$, $\tilde{A}_{ci}^T P_i P_i^{-1}$ and $P_i \tilde{D}_{i_1}$ can be easily expressed as $R^{-1} \bar{B}^T X \bar{P}_1 M_i^{\dagger}$, $\tilde{S}^T[(\tilde{A}_i^T - X_i^T \tilde{B}_i R_i^{-1} \tilde{B}_i^T) X_i] \tilde{S} M_i^{\dagger}$ and $\tilde{S}^T X_i^T \tilde{D}_i$, respectively. Then (30) is a direct consequence of what we have proved.

Thanks to

$$R_i^{-1} \tilde{B}_{i_1}^T P_i \tilde{z}_{vi}(t) = R_i^{-1} \begin{bmatrix} \tilde{B}_{i_1}^T & B_2^T \end{bmatrix} \begin{bmatrix} P_i & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{z}_{vi}(t) \\ \dot{x}_{vi_2} \end{bmatrix} = R_i^{-1} \tilde{B}_i^T X_i z_{vi}(t)$$

It follows that the expression of $\dot{u}_{vi}(t)$ in (25) can be expanded into that in (29) eventually, which completes the proof of Theorem 1.

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