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# **The preview control of a class of linear systems and its application in the fault-tolerant control theory**

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**Abstract:** In the fault-tolerant control theory based on model following control, the desired signal of the control system is the output of a reference system. This paper is concerned with the design of preview controller for a class of fault systems. A composite vector is introduced by including error vector, fault system state vector and reference system state vector. Then, we derived an augmented system from the known system equation, in which the reference input has equal status with the desired signal in the traditional preview control theory. Therefore, we can use the known theory to design the preview controller for augmented system, then the preview controller of the original fault system can be obtained by integration method. This paper strictly discusses the connection between stabilization and detectability of the augmented system and the corresponding characteristics of the original system. Finally, by applying this theory to a real steam generator water level control system, it is found that the actions of the reference input preview and the fault signal preview can effectively eliminate the effect of the fault signal on the water level of the steam generator. The simulation shows the effectiveness of the controller designed.

**Keywords:** preview control; optimal control; model following control; fault-tolerant control

## 1. Introduction

Preview control is a control method to improve the transient response of systems, suppress external disturbance and improve tracking performance by using the future information of desired signal or disturbance signal. The preview control theory is applicable to the control system that is known for the future value of the desired signal or disturbance signal. After decades of development, the preview control has basically formed a complete theoretical system (Tsuchiya & Egami, 1994). By combining with other control methods, a variety of control theories have been generated: optimal preview control (Tomizuka & Rosenthal, 1979; Liao et al., 2011), robust preview control (Kojima & Ishijima, 2003; Li & Liao, 2016), sliding mode preview control (Nonami & Ikeda, 2004; Mizuno, Saka, & Katayama, 2016), fuzzy preview control (Yeh & Tsao, 1994; Liao, Huang, & Zeng, 2010) and so on. In addition to theoretical progress, preview control has been widely applied in practical engineering fields, such as vehicle lateral control system (Peng & Tomizuka, 1993), vehicle active suspension system (Youn et al., 2017), robot system (Wang, Liu, & Chen, 2016) and other issues.

Model following control is one of the methods of active fault-tolerant control, this method does not require fault diagnosis and detection unit, when a fault occurs, the controller is designed to realize the trajectory tracking of the controlled system to the reference model with the ideal dynamic characteristics, thus obtaining the desired performance of the closed-loop system. In recent years, there are many achievements in the research of fault-tolerant control based on model following. Reference (Hu & Cheng, 1991) studies the fault-tolerant control of a class of discrete time stochastic systems and the results are applied to the aircraft model. In (Zhang & Jiang, 2002), a fault-tolerant controller with feed-forward gain is designed for the control system of a partial actuator failure. Reference (Wang et al., 2015) presents an optimal reference model based on linear quadratic optimal control method, through tracking the output of the optimal reference model, an active fault-tolerant control strategy is proposed for the control system of a large civil aircraft in the case of elevator failure. In (Bodson & Groszkiewicz, 1997; Tao, Joshi, & Ma, 2001; Zhao et al., 2014), the

design method of model reference adaptive fault-tolerant controller is given by combining model following control and adaptive control.

In the fault-tolerant control problem based on model following, there is not only a fault system, but also a fault free reference model. The reference input of the fault free reference model is a known vector (it is not the desired signal, however the output of the fault free reference model is the desired signal of the fault system), which fully meets the requirements of the preview control theory. Therefore, the combination of preview control and model following control method can be applied to the fault-tolerant control problem, which is the main purpose of this paper. The basic idea is to construct a formal system, or we can say augmented system, which combines the fault system and the reference model. The problem investigated in this article is transformed into the standard optimal preview control problem, and then the preview controller of augmented system is obtained by using the existing results in the preview control theory. The fault-tolerant preview controller is obtained after returning to the original fault system. In this paper, the proposed fault-tolerant preview control method is applied to a class of steam generator water level control system, and the numerical simulation is carried out.

## 2. Mathematical models and related assumptions

The state space equation of the fault model is considered in the theory of fault-tolerant control

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Wd(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where  $x(t) \in R^n$  is the state vector,  $u(t) \in R^r$  is the input vector,  $y(t) \in R^p$  is the output vector, and  $d(t) \in R^s$  is the known fault signal.  $A, B, W, C$  are known constant matrices with appropriate dimensions, respectively.

The fault free reference model is described by

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \\ y_m(t) = C_m x_m(t) \end{cases} \quad (2)$$

where  $x_m(t) \in R^{n_m}$ ,  $u_m(t) \in R^{r_m}$ ,  $y_m(t) \in R^p$ .  $u_m(t) \in R^{r_m}$  is a bounded vector called

reference input.

The basic problem of fault-tolerant control is to design a controller (called fault-tolerant controller) for the fault system (1) to eliminate the effects of the fault signal on the system output. In other words, the output  $y(t)$  of the closed loop system (1) can track the output  $y_m(t)$  of the fault free reference model (2) without any static error. It is noted that the dimensions of  $x_m(t)$  and  $x(t)$  can be different, and the fault signals and the reference inputs are known.

The tracking error vector is defined as the following

$$e(t) = y(t) - y_m(t) \quad (3)$$

The objective of this paper is to design a fault-tolerant controller with preview compensation for the system (1), so that the output of system (1) can track the output of reference model (2) asymptotically, namely

$$\lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} [y(t) - y_m(t)] = 0$$

We generalize the method of designing the preview controller directly for the desired signal into the control problem here. For this purpose, we construct the quadratic performance index function

$$J = \int_0^\infty [e^T(t)Q_e e(t) + u^T(t)R u(t)] dt \quad (4)$$

where  $Q_e$  and  $R$  are positive definite matrices.

It is noted that introducing the input vector's derivative  $\dot{u}(t)$  into the performance index function can make the closed loop system contain an integrator, which is helpful to eliminate static error (Liao et al., 2011).

The following assumptions are basic to the system (1) and (2).

**A1:** The coefficient matrix  $A_m$  of the system (2) is stable, namely, the eigenvalues of the  $A_m$  have negative real part.

**Remark 2.1:** When A1 is established and  $u_m(t)$  is a bounded vector, after the work of the system (2), its output  $y_m(t)$  (as the desired signal of the output

$y(t)$  of the system (1)) is characterized by maintaining both the main characteristics (only to amplify or reduce its amplitude) of the  $u_m(t)$  and the improvement of its smoothness. This point is illustrated on the condition that  $x_m(t)$  and  $u_m(t)$  are the scalar, and  $u_m(t)$  is a step signal and a periodic signal, respectively. Note that  $A_m$  and  $C_m$  are also scalars at this time.

According to (2), we have

$$y_m(t) = C_m \left\{ e^{A_m t} \left[ \int_0^t e^{-A_m s} B_m u_m(s) ds + x_m(0) \right] \right\}$$

(i)  $u_m(t)$  is a step signal, for instance, when

$$u_m(t) = \begin{cases} 0, & 0 \leq t < T \\ a, & t \geq T \end{cases},$$

where  $a$  is a constant, then we have

$$y_m(t) = \begin{cases} C_m e^{A_m t} x_m(0), & 0 \leq t < T \\ C_m \left\{ B_m e^{A_m t} \left( -\frac{a}{A_m} \right) [e^{-A_m t} - e^{-A_m T}] + e^{A_m t} x_m(0) \right\}, & t \geq T \end{cases}$$

that is,

$$y_m(t) = \begin{cases} C_m e^{A_m t} x_m(0), & 0 \leq t < T \\ C_m \left[ -\frac{a}{A_m} B_m + e^{A_m t} \left( \frac{a}{A_m} B_m e^{-A_m T} + x_m(0) \right) \right], & t \geq T \end{cases}$$

From this,  $y_m(t)$  is continuous in  $t = T$ , and due to  $A_m < 0$ , so when  $t$  is very

large,  $y_m(t) \approx -\frac{C_m a B_m}{A_m}$ .

For example, the reference input signal is selected as

$$u_m(t) = \begin{cases} 0, & 0 < t < 10 \\ 1, & t \geq 10 \end{cases}$$

and let  $A_m = -2$ ,  $B_m = 1$ ,  $C_m = 1$ ,  $x_m(0) = 1$ , then Figure 1 can be obtained.

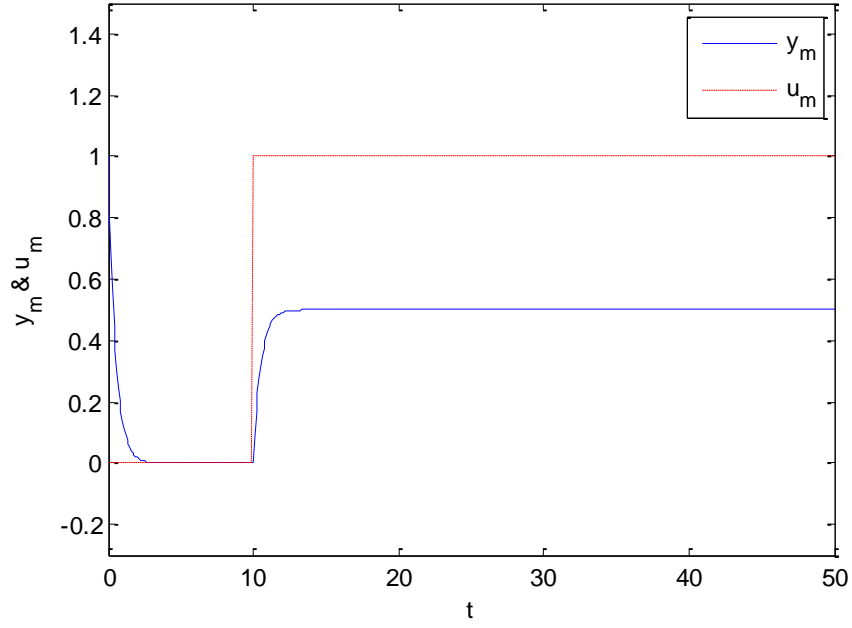


Figure 1. The graph of  $u_m(t)$  and  $y_m(t)$  ( $u_m(t)$  is a step signal)

(ii)  $u_m(t)$  is a periodic signal, for instance, when  $u_m(t) = \sin t$ , we have

$$\int_0^t u_m(s) e^{-A_m s} ds = \frac{1}{1+A_m^2} [(-\cos t - A_m \sin t) e^{-A_m t} + 1]$$

therefore,

$$y_m(t) = C_m \left\{ \frac{B_m}{1+A_m^2} (-\cos t - A_m \sin t) + \left[ \frac{B_m}{1+A_m^2} + x_m(0) \right] e^{A_m t} \right\}$$

In the same way, because  $A_m < 0$ , so  $e^{A_m t}$  is small enough when  $t$  is very large,

then

$$y_m(t) \approx \frac{C_m B_m}{1+A_m^2} (-\cos t - A_m \sin t) = -\frac{C_m B_m}{\sqrt{1+A_m^2}} \sin(t + \alpha), \text{ (where } \tan \alpha = \frac{1}{A_m} \text{)}$$

It is a periodic function of the same period with  $u_m(t) = \sin t$ , except that the amplitude and the argument are changed. Similarly, let  $A_m = -2$ ,  $B_m = 1$ ,  $C_m = 1$ ,  $x_m(0) = 1$ , Figure 2 can be obtained.

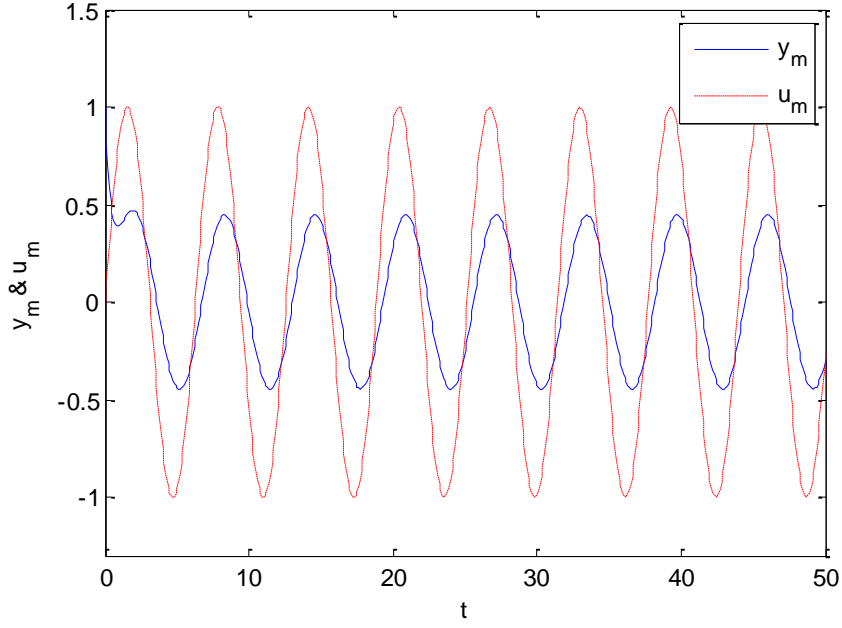


Figure 2. The graph of  $u_m(t)$  and  $y_m(t)$  ( $u_m(t)$  is a periodic signal)

**A2:** Suppose the matrices pair  $(A, B)$  is stabilizable, the matrices pair  $(C, A)$  is detectable, and the matrix  $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$  is of full row rank.

**Remark 2.2:** A2 is the basic assumption in the theory of preview control (Katayama & Hirono, 1987).

**A3:** Suppose the reference input signal  $u_m(t)$  is a piecewise continuously differentiable function satisfying

$$\lim_{t \rightarrow \infty} u_m(t) = \bar{u}_m, \quad \lim_{t \rightarrow \infty} \dot{u}_m(t) = 0$$

where  $\bar{u}_m$  is a constant vector. Moreover,  $u_m(s) (t \leq s \leq t + l_r)$  is previewable at each instant of time  $t$ ,  $l_r$  is the preview length of  $u_m(t)$ .

**A4:** Suppose the fault signal  $d(t)$  is a piecewise continuously differentiable function satisfying

$$\lim_{t \rightarrow \infty} d(t) = \bar{d}, \quad \lim_{t \rightarrow \infty} \dot{d}(t) = 0$$

where  $\bar{d}$  is a constant vector. Moreover,  $d(s) (t \leq s \leq t + l_d)$  is previewable at each



instant of time  $t$ ,  $l_d$  is the preview length of  $d(t)$ .

**Remark 2.3:** Consider the rolling system (as an automatic control system). When the sensor detects a fault (for example, the distance between rolls deviates from the normal set value) on a roll ahead, the accumulation of the billet in the fault position can be prevented by adjusting the conveying speed of the billet in rolling (Tsuchiya & Egami, 1994; Huang et al., 2004). This can be regarded as a control system with previewable fault information, in which the fault information ahead detected by the sensor is the previewable fault information.

**Remark 2.4:** A3-A4 are the basic assumptions in preview control theory (Katayama & Hirono, 1987; Liao et al., 2011).

### 3. Design of fault-tolerant preview controller

In this section, we utilize the methods of preview control theory to construct an augmented system, which combines the error equation, the state equation of fault system and the state equation of reference model.

By taking the derivative of both sides of (3), we get

$$\dot{e}(t) = \dot{y}(t) - \dot{y}_m(t) = C\dot{x}(t) - C_m\dot{x}_m(t) \quad (5)$$

Derivating both sides of the state equation in (1) and (2) gives

$$\dot{x}(t) = Ax(t) + Bu(t) + Wd(t) \quad (6)$$

$$\dot{x}_m(t) = A_mx_m(t) + B_mu_m(t) \quad (7)$$

Combining (5), (6), (7), and take  $e(t)$  as the output vector, we can obtain a form of control system (called augmented system)

$$\begin{cases} \dot{X}(t) = \bar{A}X(t) + \bar{B}u(t) + \bar{B}_m u_m(t) + \bar{W}d(t) \\ e(t) = \bar{C}X(t) \end{cases} \quad (8)$$

where

$$X(t) = \begin{bmatrix} e(t) \\ x(t) \\ x_m(t) \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} 0 & C & -C_m \\ 0 & A & 0 \\ 0 & 0 & A_m \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ B \\ 0 \end{bmatrix}, \quad \bar{B}_m = \begin{bmatrix} 0 \\ 0 \\ B_m \end{bmatrix}, \quad \bar{W} = \begin{bmatrix} 0 \\ W \\ 0 \end{bmatrix},$$

$$\bar{C} = [I \quad 0 \quad 0]$$

**Remark 3.1:** Note that the output of the system (1) is  $y(t)$ , and the output of the system (2) is  $y_m(t)$ , thus at the current time  $t$ , we can get  $e(t) = y(t) - y_m(t)$ , it is reasonable to use it as the output of the augmented system.

When the performance index function (4) is expressed by the related variables in augmented system (8), the performance index function will be adjusted as

$$J = \int_0^{\infty} \left[ X^T(t) Q X(t) + u^T(t) R u(t) \right] dt \quad (9)$$

where,  $Q = \begin{bmatrix} Q_e & & \\ & 0 & \\ & & 0 \end{bmatrix}$ .

It is known from the basic results of the optimal control theory and the structure of the performance index function (9) (that is, (4)) that the controller  $u = u(t)$  designed by the minimum value of  $J$  can make the output error  $e(t)$  of the closed loop system have the nature of the preceding description. Therefore, the problem is converted to the optimal regulation of the system (8) and the performance index function (9). Similar to the reference (Liao et al., 2011), we can obtain the following theorem.

**Theorem 1:** Suppose  $(\tilde{A}, \tilde{B})$  is stabilizable and  $(Q^{1/2}, \tilde{A})$  is detectable. The optimal preview input of system (8) under the performance index function (9) can be expressed as

$$u(t) = -R^{-1} \tilde{B}^T P X(t) - R^{-1} \tilde{B}^T g(t)$$

where  $P$  is a positive semi-definite matrix satisfying the algebraic Riccati equation

$$\tilde{A}^T P + P \tilde{A} - P \tilde{B} R^{-1} \tilde{B}^T P + Q = 0 \quad (10)$$

and

$$g(t) = \int_0^{l_d} \left[ \exp(\sigma \bar{A}_c^T) P \tilde{B}_m^T u_m(t + \sigma) \right] d\sigma + \int_0^{l_r} \left[ \exp(\sigma \bar{A}_c^T) P \tilde{B}_m^T u_m(t + \sigma) \right] d\sigma$$

the matrix in this formula

$$\bar{A}_c = \tilde{A} - \tilde{B} R^{-1} \tilde{B}^T P \quad (11)$$

is stable.

Because system (8) has the same form with system (3.4) in reference (Liao et al., 2011), their performance index functions and assumptions about preview are also in the same form. There is no difference just from the mathematical angle, so the derivation is completely similar. It is omitted here.

#### 4. Conditions for the existence of the controller

Theorem 1 requires  $(\tilde{A}, \tilde{B})$  is stabilizable and  $(Q^{\frac{1}{2}}, \tilde{A})$  is detectable, we discuss the circumstances which original system (1) needs to satisfy so that the two conditions are established. Note that the reference model (2) is a given system, so that the A1 must be satisfied.

**Lemma 1:** Under the assumption of A1,  $(\tilde{A}, \tilde{B})$  is stabilizable if and only if  $(A, B)$  is stabilizable and the matrix  $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$  is of full row rank.

**Proof:** According to the PBH criterion (Chen, 1999),  $(\tilde{A}, \tilde{B})$  is stabilizable if and only if for any complex number  $s$  satisfying  $\text{Re}(s) \geq 0$ , the matrix

$$U_c = \begin{bmatrix} -sI & C & -C_m & 0 \\ 0 & A-sI & 0 & B \\ 0 & 0 & A_m-sI & 0 \end{bmatrix}$$

is of full row rank. In the same way, use the PBH criterion, due to  $A_m$  is stable we can infer that any complex number  $s$  satisfying  $\text{Re}(s) \geq 0$ ,  $A_m - sI$  is invertible.

Therefore, when  $\text{Re}(s) \geq 0$ , we have

$$\begin{aligned} \text{rank}(U_c) &= \text{rank}(A_m - sI) + \text{rank} \begin{bmatrix} -sI & C & 0 \\ 0 & A-sI & B \end{bmatrix} \\ &= n_m + \text{rank} \begin{bmatrix} -sI & C & 0 \\ 0 & A-sI & B \end{bmatrix} \end{aligned}$$

That is to say, matrix  $U_c$  is of full row rank if and only if  $\begin{bmatrix} -sI & C & 0 \\ 0 & A-sI & B \end{bmatrix}$  is of

full row rank. Two cases of this matrix are discussed.

(i) When  $s = 0$ , obviously  $\begin{bmatrix} -sI & C & 0 \\ 0 & A-sI & B \end{bmatrix}$  is of full row rank if and only if

matrix  $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$  is of full row rank.

(ii) When  $\text{Re}(s) \geq 0$  and  $s \neq 0$ ,  $sI$  is reversible, thus  $\begin{bmatrix} -sI & C & 0 \\ 0 & A-sI & B \end{bmatrix}$  is

of full row rank if and only if matrix  $\begin{bmatrix} sI-A & B \end{bmatrix}$  is of full row rank.

Note that, from the discussion of  $s = 0$  is known that  $\begin{bmatrix} sI-A & B \end{bmatrix}|_{s=0}$  is also of full row rank, therefore, we get the conclusions to be proved by combining these discussions. This accomplishes the proof of the Lemma 1.

**Lemma 2:** Under the assumption of A1,  $(Q^{\frac{1}{2}}, \tilde{A})$  is detectable if and only if  $(C, A)$  is detectable.

**Proof:** According to the PBH criterion,  $(Q^{\frac{1}{2}}, \tilde{A})$  is detectable if and only if for any complex number  $s$  satisfying  $\text{Re}(s) \geq 0$ , the matrix  $U_o = \begin{bmatrix} A-sI \\ Q^{\frac{1}{2}} \end{bmatrix}$  is of full column rank. Note that

$$U_o = \begin{bmatrix} -sI & C & -C_m \\ 0 & A-sI & 0 \\ 0 & 0 & A_m-sI \\ Q_e^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

let  $s$  is a complex number and satisfying  $\text{Re}(s) \geq 0$ , from the structure of matrix  $U_o$  and the invertibility of matrix  $A_m-sI$  and  $Q_e^{\frac{1}{2}}$ , we have

$$\begin{aligned}
\text{rank}(U_o) &= \text{rank} \begin{bmatrix} -sI & C & -C_m \\ 0 & A-sI & 0 \\ 0 & 0 & A_m-sI \\ Q_e^{1/2} & 0 & 0 \end{bmatrix} \\
&= \text{rank}(Q_e^{1/2}) + \text{rank} \begin{bmatrix} C & -C_m \\ A-sI & 0 \\ 0 & A_m-sI \end{bmatrix} \\
&= \text{rank}(Q_e^{1/2}) + \text{rank}(A_m-sI) + \text{rank} \begin{bmatrix} A-sI \\ C \end{bmatrix} \\
&= p + n_m + \text{rank} \begin{bmatrix} A-sI \\ C \end{bmatrix}
\end{aligned}$$

From this, and citing the PBH criterion again, it can be proved that Lemma 2 is established.

Theorem 1 is the result of using optimal control theory for augmented system. Lemma 1 and Lemma 2 show the relationship between the stabilization and detectability of augmented system and the corresponding characteristics of the original system. Now we need to go back to the original system to get the controller we required. Using the Lemmas 1 and 2 in Theorem 1 and solving the  $u(t)$ , we get the final result of this paper.

**Theorem 2:** If A1, A2, A3 and A4 hold,  $Q_e$  and  $R$  are positive definite matrices, then, the Riccati equation (10) has a unique symmetric positive semi-definite solution, and the optimal preview input of system (1) under the performance index function (4) can be expressed as

$$u(t) = u(0) - K_e \int_0^t e(\sigma) d\sigma - K_x [x(t) - x(0)] - K_{x_m} [x_m(t) - x_m(0)] + f_1(t) + f_2(t) \quad (12)$$

where  $f_1(t)$ ,  $f_2(t)$ ,  $K_e$ ,  $K_x$  and  $K_{x_m}$  are

$$\begin{aligned}
f_1(t) &= -R^{-1} B^T \int_0^{l_d} \exp(\sigma \bar{A}_c^T) P W [d(t+\sigma) - d(\sigma)] d\sigma \\
f_2(t) &= -R^{-1} B^T \int_0^{l_r} \exp(\sigma \bar{A}_c^T) P B_m^T [u_m(t+\sigma) - u_m(\sigma)] d\sigma \\
K &= R^{-1} B^T P = \begin{bmatrix} K_e & K_x & K_{x_m} \end{bmatrix} \quad (13)
\end{aligned}$$

stability matrix  $\bar{A}_c$  is shown as (11).  $u(0)$ ,  $x(0)$  and  $x_m(0)$  are the initial values

that can be arbitrarily taken.

**Proof:** It can be obtained by utilizing (13)

$$u(t) = -K_e e(t) - K_x x(t) - K_{x_m} x_m(t) - R^{-1} B^T g(t)$$

integrating on  $[0, t]$  in both sides, we get

$$u(t) = u(0) - K_e \int_0^t e(\sigma) d\sigma - K_x [x(t) - x(0)] - K_{x_m} [x_m(t) - x_m(0)] - R^{-1} B^T \int_0^t g(s) ds \quad (14)$$

Due to calculation

$$\begin{aligned} \int_0^t g(s) ds &= \int_0^t \int_0^{l_d} \left[ \exp(\sigma \bar{A}_c^T) P W \bar{u}(s + \sigma) \right] d\sigma ds + \int_0^t \int_0^{l_r} \left[ \exp(\sigma \bar{A}_c^T) P \bar{B}_m \bar{u}_m(s + \sigma) \right] d\sigma ds \\ &= \int_0^{l_d} \exp(\sigma \bar{A}_c^T) P W \left[ \int_0^t \bar{u}(s + \sigma) ds \right] d\sigma + \int_0^{l_r} \exp(\sigma \bar{A}_c^T) P \bar{B}_m \left[ \int_0^t \bar{u}_m(s + \sigma) ds \right] d\sigma \\ &= \int_0^{l_d} \exp(\sigma \bar{A}_c^T) P W [d(t + \sigma) - d(\sigma)] d\sigma + \int_0^{l_r} \exp(\sigma \bar{A}_c^T) P \bar{B}_m [u_m(t + \sigma) - u_m(\sigma)] d\sigma \end{aligned}$$

substituting this equation into (14) yields the conclusion to be proved.

Notice that the control effect can be improved by selecting the initial values  $x(0)$ ,  $x_m(0)$  and  $u(0)$ .

**Remark 4.1:** In (12),  $-K_e \int_0^t e(\sigma) d\sigma$  is the integral of the output tracking error, that is, the integrator;  $-K_x x(t)$  is the state feedback of the fault model (1);  $-K_{x_m} x_m(t)$  is the effect of the fault free reference model (2) on the closed loop system;  $f_1(t)$  and  $f_2(t)$  are the preview compensation of the fault signal and reference input signal, respectively.

The controller given by (12) is called fault-tolerant preview controller.

## 5. Numerical simulation

**Example:** The dynamic equation of a real steam generator water level control system (Irving, Miossec, & Tassart, 1980) is

$$y(s) = \frac{G_1}{s} (q_e(s) - q_v(s)) - \frac{G_2}{1 + \tau_2 s} (q_e(s) - q_v(s)) + \frac{G_3 s}{\tau_1^{-2} + 4\pi^2 T^{-2} + 2\tau_1^{-1} s + s^2} q_e(s)$$

The corresponding state space equation can be described by (1), where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 1 \end{bmatrix}, \quad W = \begin{bmatrix} w_1 \\ w_2 \\ 0 \\ 0 \end{bmatrix}, \quad C = [1 \quad 1 \quad 0 \quad G_3],$$

$$\begin{cases} a_{22} = -\tau_2^{-1}, & a_{43} = -(\tau_1^{-2} + 4\pi^2 T^{-2}), & a_{44} = -2\tau_1^{-1} \\ b_1 = G_1, & b_2 = -G_2\tau_2^{-1} \\ w_1 = -G_1, & w_2 = G_2\tau_2^{-1} \end{cases}.$$

By selecting a set of parameters with a power of 5%, the coefficient matrices in (15) is obtained

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -0.02066 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.00333 & -0.04773 \end{bmatrix}, \quad B = \begin{bmatrix} 0.058 \\ -0.19897 \\ 0 \\ 1 \end{bmatrix}, \quad W = \begin{bmatrix} -0.058 \\ 0.19897 \\ 0 \\ 0 \end{bmatrix},$$

$$C = [1 \quad 1 \quad 0 \quad 0.181].$$

In this system, the input vector  $u(t)$  is the feedwater flow of the steam generator, and the disturbance vector  $d(t)$  is a known fault signal. The purpose of water level control is to eliminate the effects of fault signal  $d(t)$  on the water level  $y(t)$  of the steam generator by adjusting the flow rate of feedwater  $u(t)$ . In other words, when the fault occurs, the water level  $y(t)$  of the steam generator can be kept at the ideal water level by the design of the controller  $u(t)$ .

Suppose in the reference model (2)

$$A_m = \begin{bmatrix} -2 & 0.2 \\ 0 & -0.7 \end{bmatrix}, \quad B_m = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_m = [1 \quad 1]$$

and the reference input is a step signal

$$u_m(t) = \begin{cases} 0, & 0 \leq t \leq 75 \\ 10, & t > 75 \end{cases}$$

Note that the output vector  $y_m(t)$  of (2) is the ideal water level of the steam generator.

Then, suppose the fault signal is

$$d(t) = \begin{cases} 0, & 0 \leq t \leq 350 \\ 2, & t > 350 \end{cases}$$

The weight matrices of the performance index function (4) are taken as  $Q_e = 0.009$ ,  $R = 5000$ .

It is easy to verify that the conditions of theorem 2 are all satisfied. The solution of the Riccati equation and the gain matrix in the fault-tolerant preview controller (12) are

$$P = \begin{bmatrix} 3.170643 & 558.4987 & 131.4901 & -0.55135 & 0.477859 & -1.58307 & -4.96159 \\ 558.4987 & 128707 & 25654.74 & -101.53 & 2.77573 & -279.248 & -877.611 \\ 131.4901 & 25654.74 & 5810.079 & -23.8474 & 6.491519 & -65.7439 & -206.598 \\ -0.55135 & -101.53 & -23.8474 & 0.100913 & -0.04224 & 0.275675 & 0.866385 \\ 0.477859 & 2.77573 & 6.491519 & -0.04224 & 0.926958 & -0.23873 & -0.74585 \\ -1.58307 & -279.248 & -65.7439 & 0.275675 & -0.23873 & 0.790973 & 2.479922 \\ -4.96159 & -877.611 & -206.598 & 0.866385 & -0.74585 & 2.479922 & 7.780668 \end{bmatrix}$$

$$K_e = 0.0013416,$$

$$K_x = [0.47265 \quad 0.067687 \quad -0.00023721 \quad -0.000040733],$$

$$K_{x_m} = [-0.00067081 \quad -0.0021081],$$

where  $P$  is positive definite.

The initial conditions are as follows:

$$u(0) = 0.1, \quad x(0) = \begin{bmatrix} 0.75 \\ 0 \\ -0.75 \\ 0.3 \end{bmatrix}, \quad x_m(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Figure 3 shows the output response of the water level control system when the preview length of the fault signal  $d(t)$  is  $l_d = 0$  (i.e., without fault signal preview). Figure 4 and Figure 5 show the corresponding tracking error and input signal, respectively.



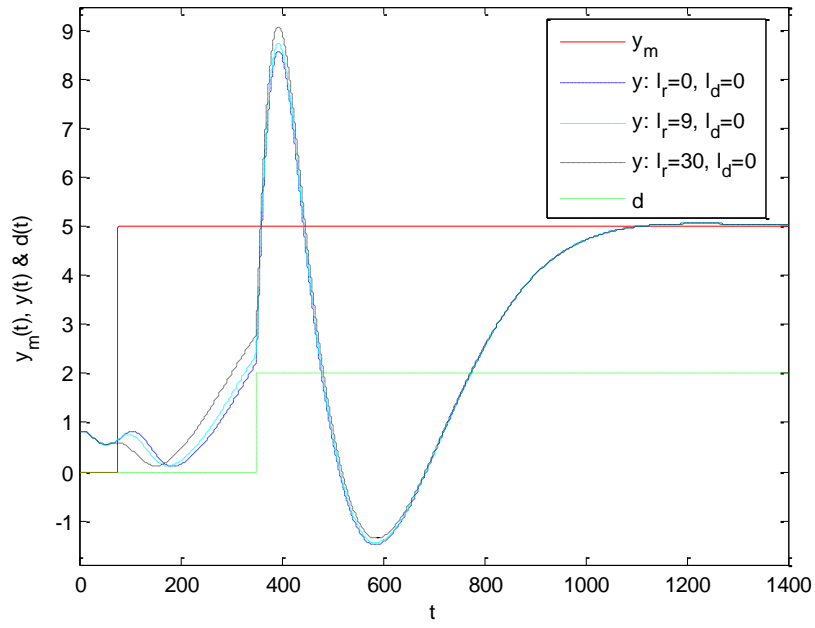


Figure 3. The output response of the water level control system without fault signal  
preview

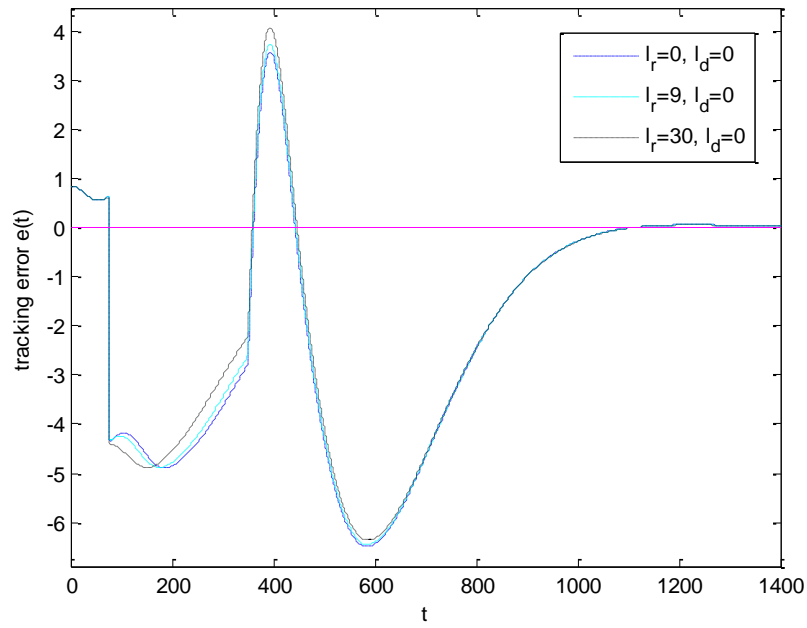


Figure 4. The output error of the water level control system without fault signal  
preview

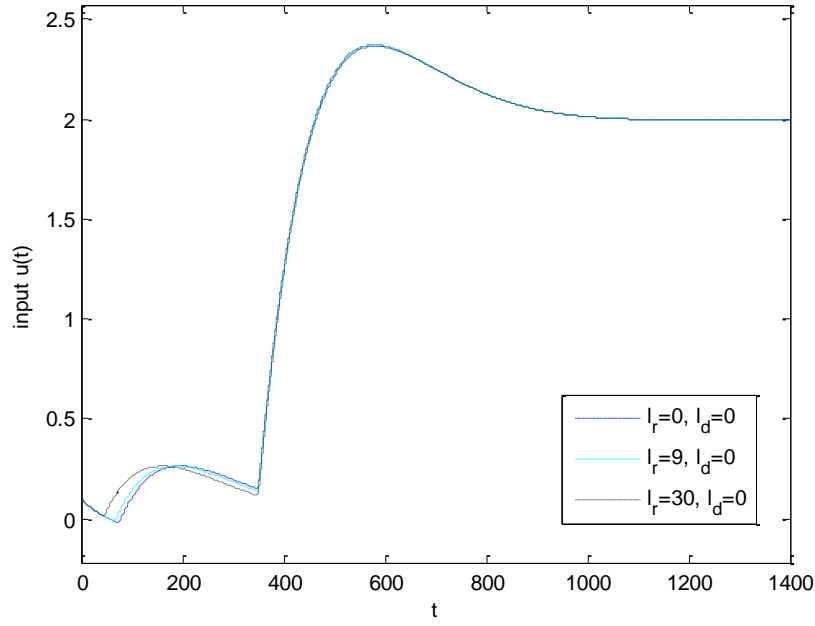


Figure 5. The input of the water level control system without fault signal preview

From Figure 3 and Figure 4, it can be seen that the existence of the preview compensation accelerates the response speed of the system back to the ideal value. In fact, after calculation, we can know that beginning around  $t = 450$ , the tracking error is largest when  $l_r = 0$ , the tracking error is smaller when  $l_r = 9$ , and it is smallest when  $l_r = 30$  which indicates the effect of the effect about  $u_m(t)$  preview. In the same way, Figure 3 and Figure 4 show that the increase of the preview length of  $u_m(t)$  may increase the overshoot in short time when the fault signal is added. These characteristics need to be paid attention to when they are applied, and a proper preview length should be taken. Figure 5 illustrates that once the system overcomes the effects of the fault signal back to the ideal value, the input vector is basically maintained, which is exactly the same as the actual situation.

Figures 6, 7 and 8 reveal the output response, the corresponding tracking error and input signal of the water level control system when the preview length of the reference input signal  $u_m(t)$  is  $l_r = 0$  (i.e., reference input signal preview), respectively. It can also be noticed that the preview compensation of the fault signal has an effect on the ideal value of the water level recovery. The addition of preview

reduces the adverse effects caused by the fault signal, improves the accuracy of the water level tracking ideal value, and the longer the preview length is, the more obvious the effect is. In particular, it is evident from Figure 6 and Figure 7 that the effect of the fault on the change of the water level is effectively suppressed because of the addition of the fault signal preview. Moreover, when the preview length of the fault signal is increased, the adjustment time of the system can be shortened and the overshoot is reduced.

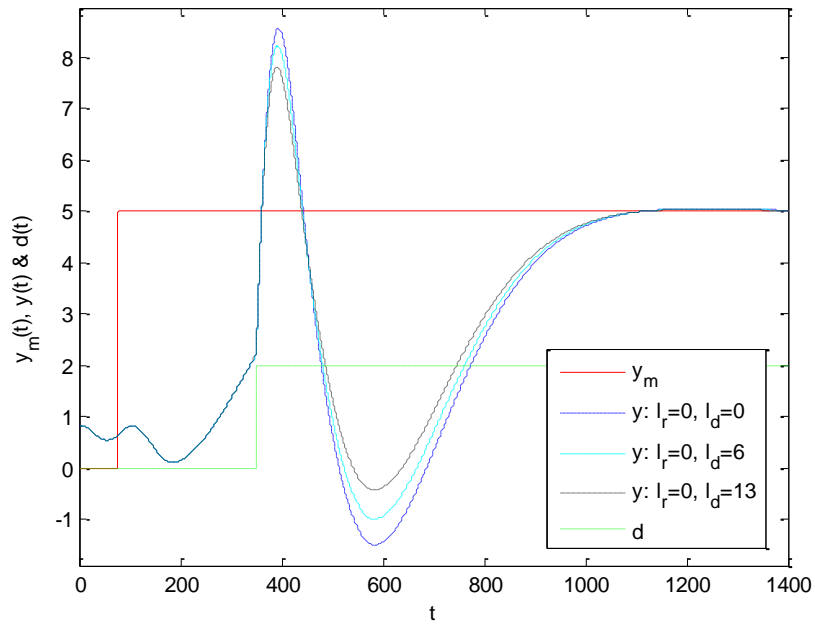


Figure 6. The output response of the water level control system without reference input signal preview

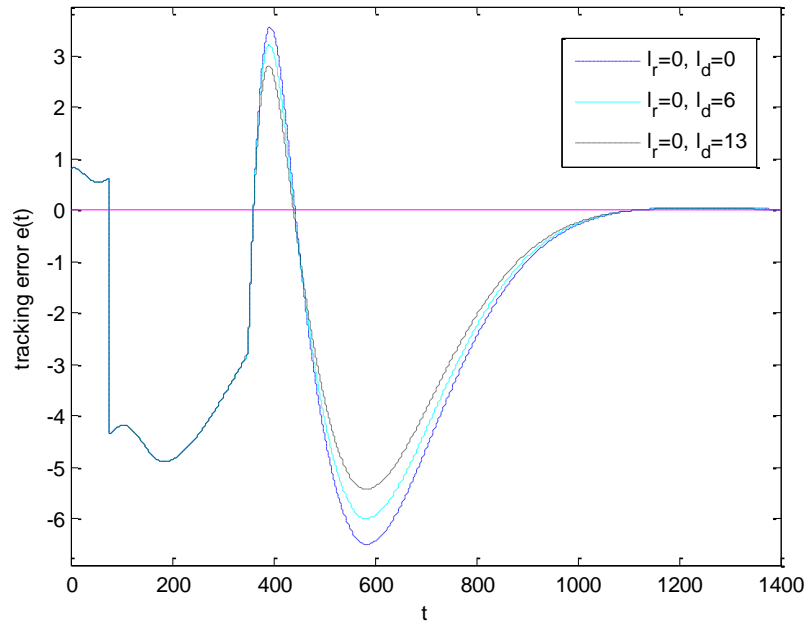


Figure 7. The output error of the water level control system without reference input signal preview

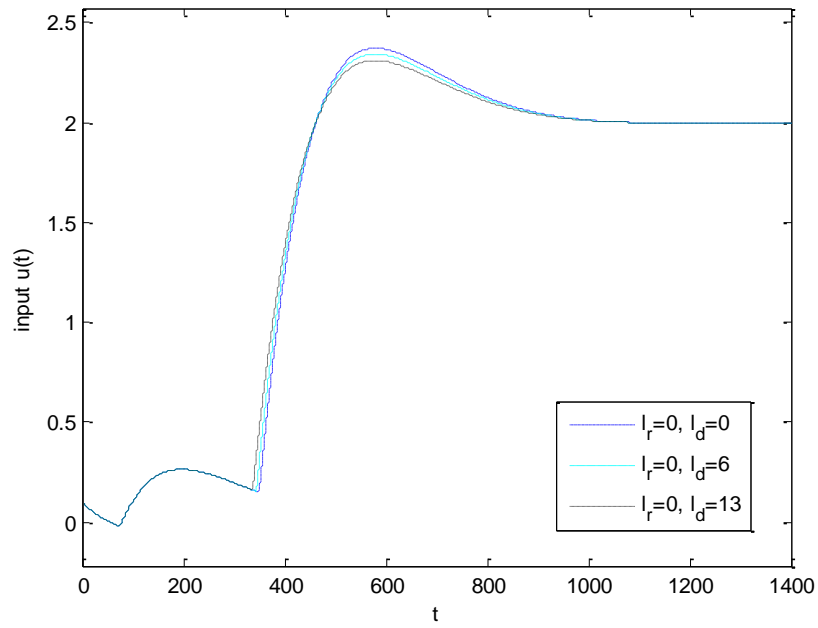


Figure 8. The input of the water level control system without reference input signal preview

Figures 9, 10 and 11 demonstrate the output response, the corresponding tracking error and input signal of the water level control system with the preview of the reference input signal and fault signal, respectively. Similarly, the longer the preview

length is, the faster the system will overcome the fault and return to the ideal value.

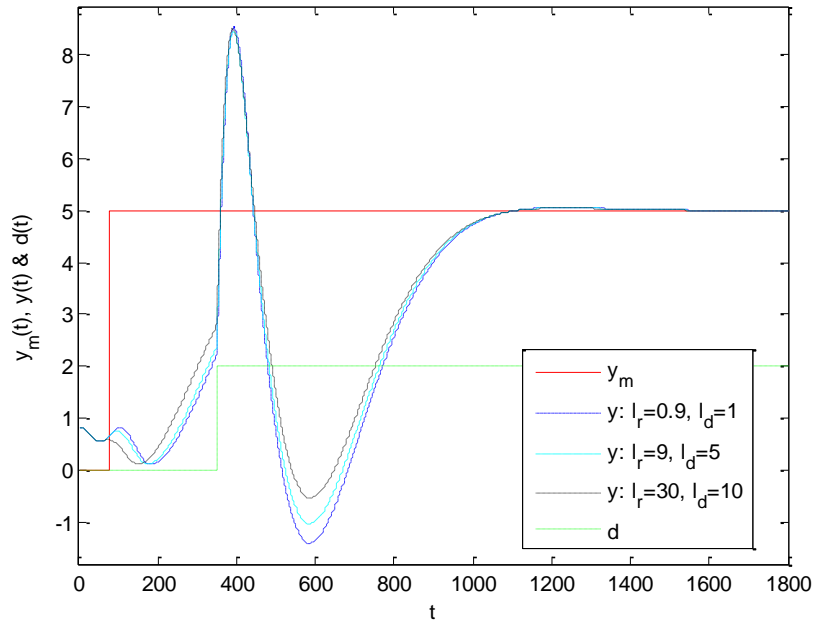


Figure 9. The output response of the water level control system with the preview of the reference input signal and fault signal

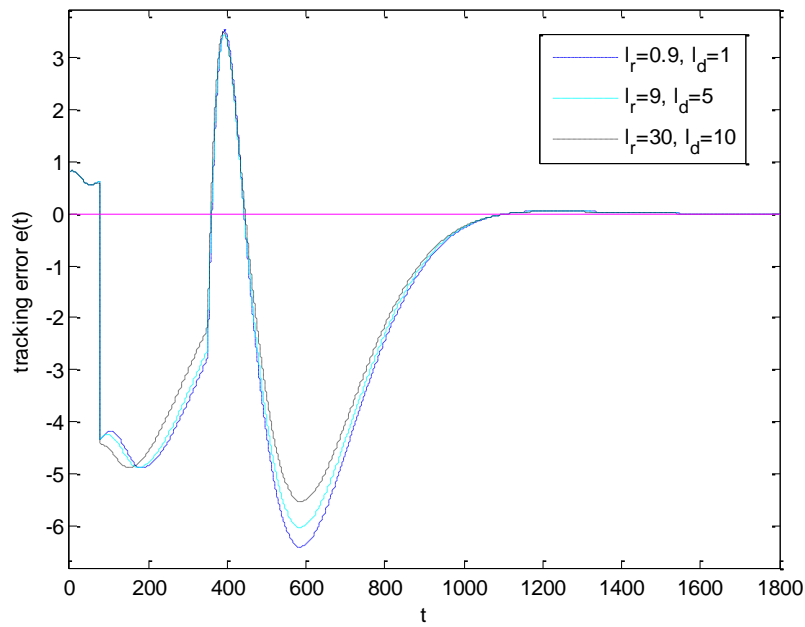


Figure 10. The output error of the water level control system with the preview of the reference input signal and fault signal

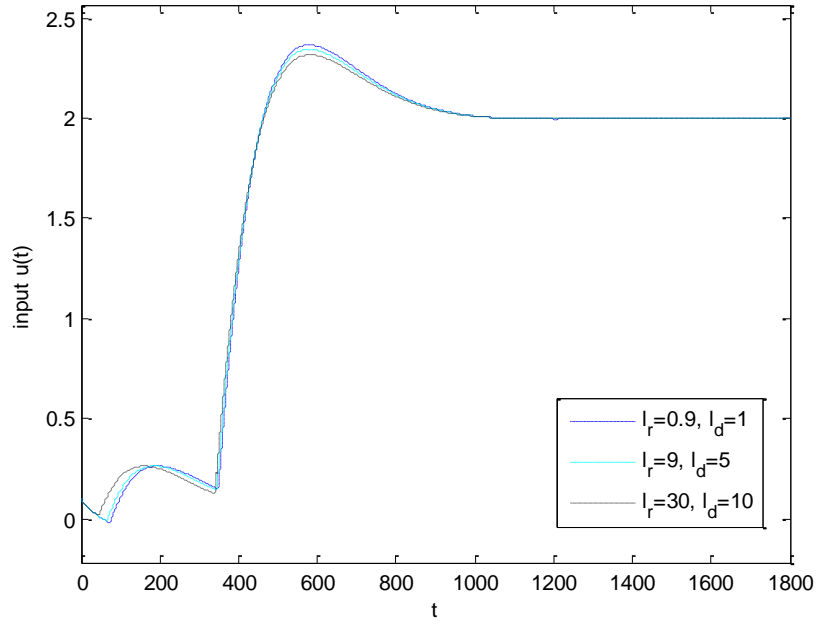


Figure 11. The input of the water level control system with the preview of the reference input signal and fault signal

It should be pointed out that if the fault signal is 0 (i.e., there is no fault signal), the controller designed here is still valid. Figures 12, 13 and 14 show the output response, the corresponding tracking error, and input signal of the water level control system in such condition, respectively. The simulation results show that the effect of water level recovery is noticeable under the reference input preview compensation.

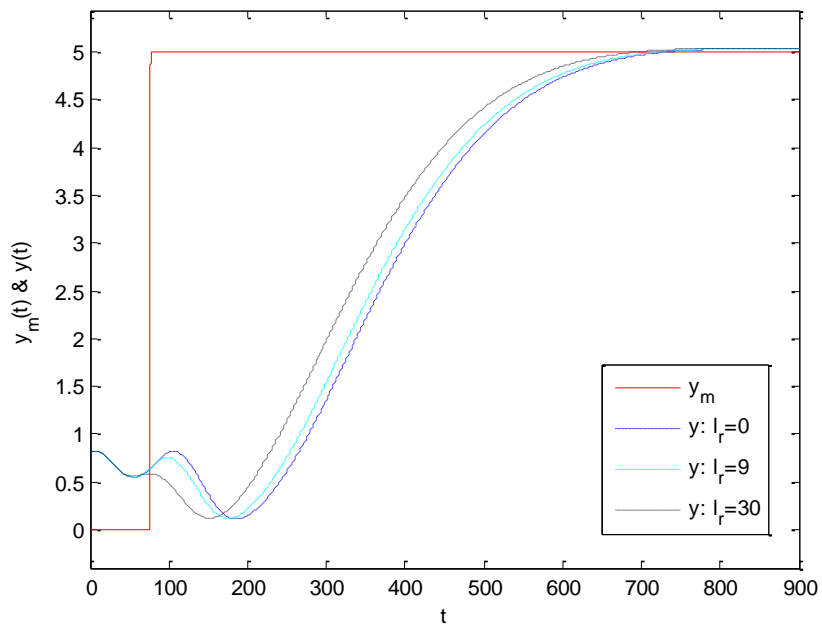


Figure 12. The output response of the water level control system without fault signal

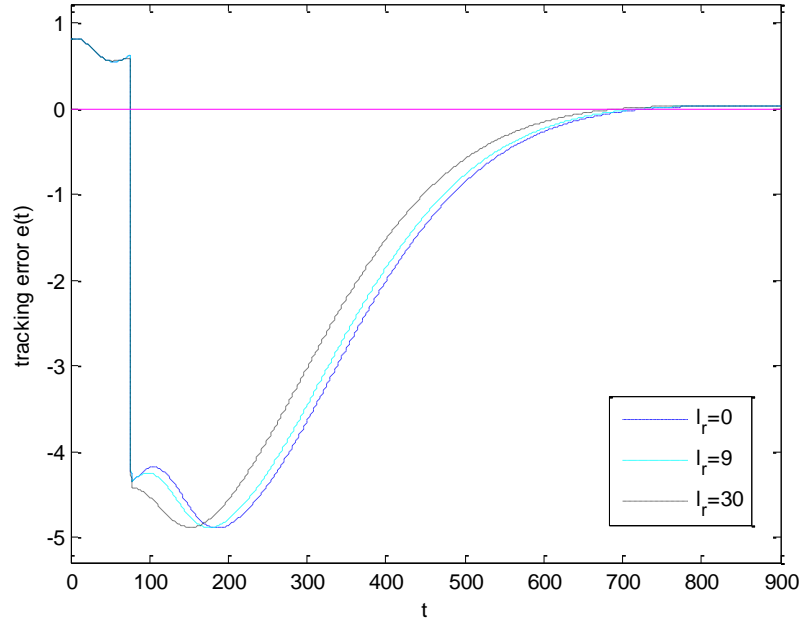


Figure 13. The output error of the water level control system without fault signal

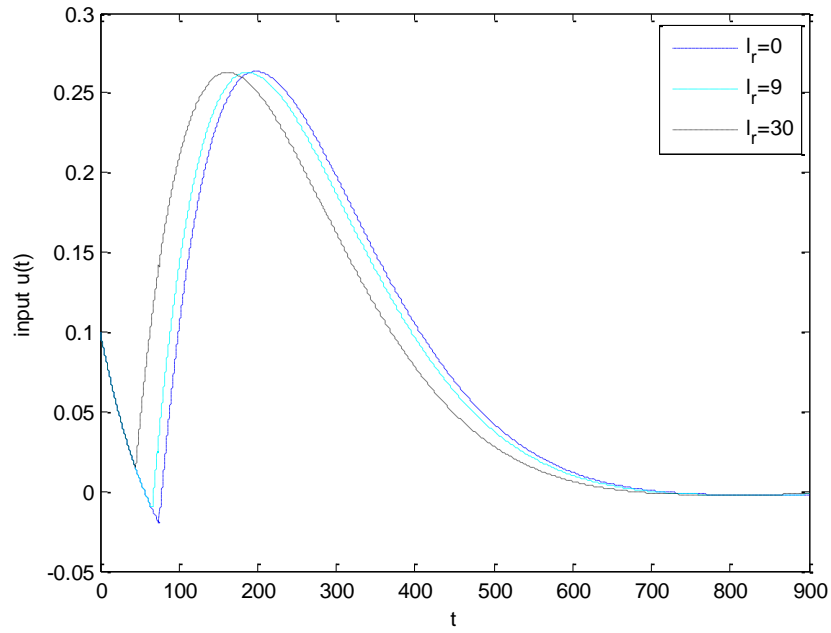


Figure 14. The input of the water level control system without fault signal

**Remark 5.1:** The controller designed in this paper has two main advantages: firstly, the existence of the preview compensation accelerates the response speed of the system; secondly, the designed control contains integrator, which can eliminate static error(Liao et al. , 2003).

## 6. Conclusion

In this paper, the theory of the preview control is developed by combining with model-following control, and which is successfully applied to the fault-tolerant problem. The fault-tolerant preview controller for a class of fault systems is designed. Apart from the disturbance preview, it is different from the traditional preview control that the desired signal of the system is the output of the reference model and the previewable signal is the input of the reference model. By constructing augmented system, the problem is transformed into the preview controller design problem of augmented system. Then, the preview controller of the augmented system is obtained by using the similar research method in the preview control problem, from which it can be gained the required fault-tolerant preview controller. Finally, the results are applied to a steam generator water level control system, and the numerical simulation is carried out to illustrate the effectiveness of the proposed method.

In future, one of the important research topics is to extend the present result to the continuous time linear descriptor systems, which is more challenging due to the special structure of descriptor systems. And the issue of the fault-tolerant preview control based on the techniques of sliding mode will be interesting and of great significance.

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