The preview control of a class of linear systems and its application in the fault-tolerant control theory

Fucheng Liao[a,b*] Chen Jia[a,b] Usman Malik[a] Xiao Yu[a,b] Jiamei Deng[c]

[a]Department of Information and Computing Science, School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China;

[b]Beijing Key Laboratory for Magneto-Photoelectrical Composite and Interface Science, School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China;

[c]School of Computing, Creative Technologies, and Engineering, Leeds Beckett University, Leeds LS1 3HE, UK

* Corresponding author. Email: fcliao@ustb.edu.cn

Abstract: In the fault-tolerant control theory based on model following control, the desired signal of the control system is the output of a reference system. This paper is concerned with the design of preview controller for a class of fault systems. A composite vector is introduced by including error vector, fault system state vector and reference system state vector. Then, we derived an augmented system from the known system equation, in which the reference input has equal status with the desired signal in the traditional preview control theory. Therefore, we can use the known theory to design the preview controller for augmented system, then the preview controller of the original fault system can be obtained by integration method. This paper strictly discusses the connection between stabilization and detectability of the augmented system and the corresponding characteristics of the original system. Finally, by applying this theory to a real steam generator water level control system, it is found that the actions of the reference input preview and the fault signal preview can effectively eliminate the effect of the fault signal on the water level of the steam generator. The simulation shows the effectiveness of the controller designed.

Keywords: preview control; optimal control; model following control; fault-tolerant control

1. Introduction

Preview control is a control method to improve the transient response of systems, suppress external disturbance and improve tracking performance by using the future information of desired signal or disturbance signal. The preview control theory is applicable to the control system that is known for the future value of the desired signal or disturbance signal. After decades of development, the preview control has basically formed a complete theoretical system (Tsuchiya & Egami, 1994). By combining with other control methods, a variety of control theories have been generated: optimal preview control (Tomizuka & Rosenthal, 1979; Liao et al., 2011), robust preview control (Kojima & Ishijima, 2003; Li & Liao, 2016), sliding mode preview control (Nonami & Ikedo, 2004; Mizuno, Saka, & Katayama, 2016), fuzzy preview control (Yeh & Tsao, 1994; Liao, Huang, & Zeng, 2010) and so on. In addition to theoretical progress, preview control has been widely applied in practical engineering fields, such as vehicle lateral control system (Peng & Tomizuka, 1993), vehicle active suspension system (Youn et al., 2017), robot system (Wang, Liu, & Chen, 2016) and other issues.

Model following control is one of the methods of active fault-tolerant control, this method does not require fault diagnosis and detection unit, when a fault occurs, the controller is designed to realize the trajectory tracking of the controlled system to the reference model with the ideal dynamic characteristics, thus obtaining the desired performance of the closed-loop system. In recent years, there are many achievements in the research of fault-tolerant control based on model following. Reference (Hu & Cheng, 1991) studies the fault-tolerant control of a class of discrete time stochastic systems and the results are applied to the aircraft model. In (Zhang & Jiang, 2002), a fault-tolerant controller with feed-forward gain is designed for the control system of a partial actuator failure. Reference (Wang et al., 2015) presents an optimal reference model based on linear quadratic optimal control method, through tracking the output of the optimal reference model, an active fault-tolerant control strategy is proposed for the control system of a large civil aircraft in the case of elevator failure. In (Bodson & Groszkiewicz, 1997; Tao, Joshi, & Ma, 2001; Zhao et al., 2014), the design method of model reference adaptive fault-tolerant controller is given by combining model following control and adaptive control.

In the fault-tolerant control problem based on model following, there is not only a fault system, but also a fault free reference model. The reference input of the fault free reference model is a known vector (it is not the desired signal, however the output of the fault free reference model is the desired signal of the fault system), which fully meets the requirements of the preview control theory. Therefore, the combination of preview control and model following control method can be applied to the fault-tolerant control problem, which is the main purpose of this paper. The basic idea is to construct a formal system, or we can say augmented system, which combines the fault system and the reference model. The problem investigated in this article is transformed into the standard optimal preview control problem, and then the preview controller of augmented system is obtained by using the existing results in the preview control theory. The fault-tolerant preview controller is obtained after returning to the original fault system. In this paper, the proposed fault-tolerant preview control method is applied to a class of steam generator water level control system, and the numerical simulation is carried out.

2. Mathematical models and related assumptions

The state space equation of the fault model is considered in the theory of fault-tolerant control

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Wd(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the input vector, $y(t) = R^p$ is the output vector, and $d(t) \in R^s$ is the known fault signal. A, B, W, C are known constant matrices with appropriate dimensions, respectively.

The fault free reference model is described by

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \\ y_m(t) = C_m x_m(t) \end{cases} \tag{2}$$

where $x_m(t) \in R^{n_m}$, $u_m(t) \in R^{r_m}$, $y_m(t) = R^p$. $u_m(t) \in R^{r_m}$ is a bounded vector called reference input.

The basic problem of fault-tolerant control is to design a controller (called fault-tolerant controller) for the fault system (1) to eliminate the effects of the fault signal on the system output. In other words, the output $y(t)$ of the closed loop system (1) can track the output $y_m(t)$ of the fault free reference model (2) without any static error. It is noted that the dimensions of $x_m(t)$ and $x(t)$ can be different, and the fault signals and the reference inputs are known.

The tracking error vector is defined as the following

$$e(t) = y(t) - y_m(t) \tag{3}$$

The objective of this paper is to design a fault-tolerant controller with preview compensation for the system (1), so that the output of system (1) can track the output of reference model (2) asymptotically, namely

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}[y(t) - y_m(t)] = 0$$

We generalize the method of designing the preview controller directly for the desired signal into the control problem here. For this purpose, we construct the quadratic performance index function

$$J = \int_0^\infty \left[e^T(t)Q_e e(t) + \dot{u}^T(t)R\dot{u}(t) \right] dt \tag{4}$$

where Q_e and R are positive definite matrices.

It is noted that introducing the input vector's derivative $\dot{u}(t)$ into the performance index function can make the closed loop system contain an integrator, which is helpful to eliminate static error (Liao et al., 2011).

The following assumptions are basic to the system (1) and (2).

A1: The coefficient matrix A_m of the system (2) is stable, namely, the eigenvalues of the A_m have negative real part.

Remark 2.1: When A1 is established and $u_m(t)$ is a bounded vector, after the work of the system (2), its output $y_m(t)$ (as the desired signal of the output

$y(t)$ of the system (1)) is characterized by maintaining both the main characteristics (only to amplify or reduce its amplitude) of the $u_m(t)$ and the improvement of its smoothness. This point is illustrated on the condition that $x_m(t)$ and $u_m(t)$ are the scalar, and $u_m(t)$ is a step signal and a periodic signal, respectively. Note that A_m and C_m are also scalars at this time.

According to (2), we have

$$y_m(t) = C_m \left\{ e^{A_m t} [\int_0^t e^{-A_m s} B_m u_m(s) ds + x_m(0)] \right\}$$

(i) $u_m(t)$ is a step signal, for instance, when

$$u_m(t) = \begin{cases} 0, & 0 \le t < T \\ a, & t \ge T \end{cases},$$

where a is a constant, then we have

$$y_m(t) = \begin{cases} C_m e^{A_m t} x_m(0), & 0 \le t < T \\ C_m \left\{ B_m e^{A_m t} \left(-\dfrac{a}{A_m} \right) [e^{-A_m t} - e^{-A_m T}] + e^{A_m t} x_m(0) \right\}, & t \ge T \end{cases}$$

that is,

$$y_m(t) = \begin{cases} C_m e^{A_m t} x_m(0), & 0 \le t < T \\ C_m \left[-\dfrac{a}{A_m} B_m + e^{A_m t} \left(\dfrac{a}{A_m} B_m e^{-A_m T} + x_m(0) \right) \right], & t \ge T \end{cases}$$

From this, $y_m(t)$ is continuous in $t = T$, and due to $A_m < 0$, so when t is very large, $y_m(t) \approx -\dfrac{C_m a B_m}{A_m}$.

For example, the reference input signal is selected as

$$u_m(t) = \begin{cases} 0, & 0 < t < 10 \\ 1, & t \ge 10 \end{cases}$$

and let $A_m = -2$, $B_m = 1$, $C_m = 1$, $x_m(0) = 1$, then Figure 1 can be obtained.

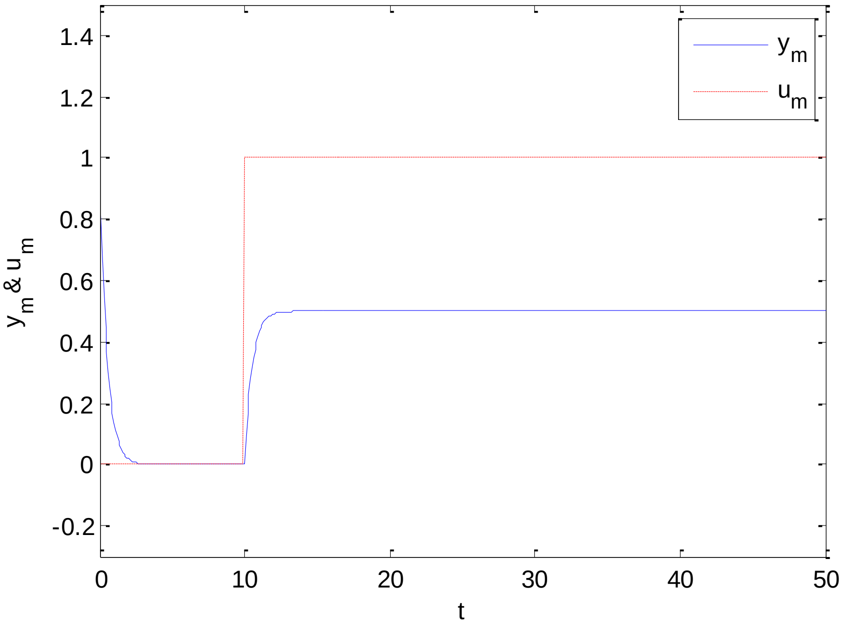

Figure 1. The graph of $u_m(t)$ and $y_m(t)$ ($u_m(t)$ is a step signal)

(ii) $u_m(t)$ is a periodic signal, for instance, when $u_m(t)=\sin t$, we have

$$\int_0^t u_m(s)e^{-A_m s}ds=\frac{1}{1+A_m^2}[(-\cos t-A_m\sin t)e^{-A_m t}+1]$$

therefore,

$$y_m(t)=C_m\left\{\frac{B_m}{1+A_m^2}(-\cos t-A_m\sin t)+\left[\frac{B_m}{1+A_m^2}+x_m(0)\right]e^{A_m t}\right\}$$

In the same way, because $A_m<0$, so $e^{A_m t}$ is small enough when t is very large, then

$$y_m(t)\approx\frac{C_m B_m}{1+A_m^2}(-\cos t-A_m\sin t)=-\frac{C_m B_m}{\sqrt{1+A_m^2}}\sin(t+\alpha)\text{, (where } \tan\alpha=\frac{1}{A_m}\text{)}$$

It is a periodic function of the same period with $u_m(t)=\sin t$, except that the amplitude and the argument are changed. Similarly, let $A_m=-2$, $B_m=1$, $C_m=1$, $x_m(0)=1$, Figure 2 can be obtained.

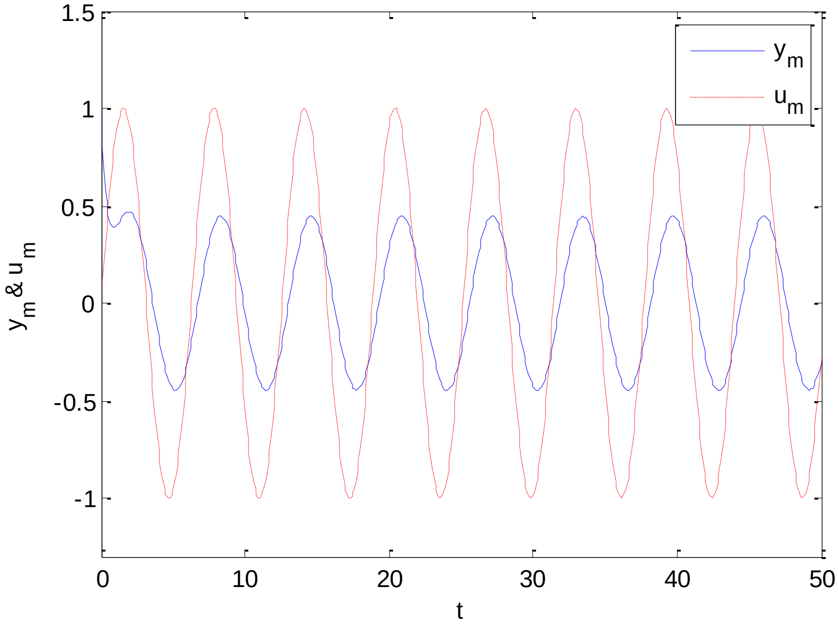

Figure 2. The graph of $u_m(t)$ and $y_m(t)$ ($u_m(t)$ is a periodic signal)

A2: Suppose the matrices pair (A, B) is stabilizable, the matrices pair (C, A) is detectable, and the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank.

Remark 2.2: A2 is the basic assumption in the theory of preview control (Katayama & Hirono, 1987).

A3: Suppose the reference input signal $u_m(t)$ is a piecewise continuously differentiable function satisfying

$$\lim_{t\to\infty} u_m(t) = \bar{u}_m, \quad \lim_{t\to\infty} \dot{u}_m(t) = 0$$

where $\bar{u}_m$ is a constant vector. Moreover, $u_m(s)\,(t \le s \le t + l_r)$ is previewable at each instant of time t, l_r is the preview length of $u_m(t)$.

A4: Suppose the fault signal $d(t)$ is a piecewise continuously differentiable function satisfying

$$\lim_{t\to\infty} d(t) = \bar{d}, \quad \lim_{t\to\infty} \dot{d}(t) = 0$$

where $\bar{d}$ is a constant vector. Moreover, $d(s)\,(t \le s \le t + l_d)$ is previewable at each instant of time t, l_d is the preview length of $d(t)$.

Remark 2.3: Consider the rolling system (as an automatic control system). When the sensor detects a fault (for example, the distance between rolls deviates from the normal set value) on a roll ahead, the accumulation of the billet in the fault position can be prevented by adjusting the conveying speed of the billet in rolling (Tsuchiya & Egami, 1994; Huang et al., 2004). This can be regarded as a control system with previewable fault information, in which the fault information ahead detected by the sensor is the previewable fault information.

Remark 2.4: A3-A4 are the basic assumptions in preview control theory (Katayama & Hirono, 1987; Liao et al., 2011).

3. Design of fault-tolerant preview controller

In this section, we utilize the methods of preview control theory to construct an augmented system, which combines the error equation , the state equation of fault system and the state equation of reference model.

By taking the derivative of both sides of (3), we get

$$\dot{e}(t) = \dot{y}(t) - \dot{y}_m(t) = C\dot{x}(t) - C_m\dot{x}_m(t) \tag{5}$$

Derivating both sides of the state equation in (1) and (2) gives

$$\ddot{x}(t) = A\dot{x}(t) + B\dot{u}(t) + W\dot{d}(t) \tag{6}$$

$$\ddot{x}_m(t) = A_m\dot{x}_m(t) + B_m\dot{u}_m(t) \tag{7}$$

Combining (5), (6), (7), and take $e(t)$ as the output vector, we can obtain a form of control system (called augmented system)

$$\begin{cases} \dot{X}(t) = \tilde{A}X(t) + \tilde{B}\dot{u}(t) + \tilde{B}_m\dot{u}_m(t) + \tilde{W}\dot{d}(t) \\ e(t) \;= \tilde{C}X(t) \end{cases} \tag{8}$$

where

$$X(t) = \begin{bmatrix} e(t) \\ \dot{x}(t) \\ \dot{x}_m(t) \end{bmatrix},\quad \tilde{A} = \begin{bmatrix} 0 & C & -C_m \\ 0 & A & 0 \\ 0 & 0 & A_m \end{bmatrix},\quad \tilde{B} = \begin{bmatrix} 0 \\ B \\ 0 \end{bmatrix},\quad \tilde{B}_m = \begin{bmatrix} 0 \\ 0 \\ B_m \end{bmatrix},\quad \tilde{W} = \begin{bmatrix} 0 \\ W \\ 0 \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} I & 0 & 0 \end{bmatrix}$$

Remark 3.1: Note that the output of the system (1) is $y(t)$, and the output of the system (2) is $y_m(t)$, thus at the current time t, we can get $e(t) = y(t) - y_m(t)$, it is reasonable to use it as the output of the augmented system.

When the performance index function (4) is expressed by the related variables in augmented system (8), the performance index function will be adjusted as

$$J = \int_0^{\infty} \left[X^T(t) Q X(t) + \dot{u}^T(t) R \dot{u}(t) \right] dt \tag{9}$$

where, $Q = \begin{bmatrix} Q_e & & \\ & 0 & \\ & & 0 \end{bmatrix}$.

It is known from the basic results of the optimal control theory and the structure of the performance index function (9) (that is, (4)) that the controller $u = u(t)$ designed by the minimum value of J can make the output error $e(t)$ of the closed loop system have the nature of the preceding description. Therefore, the problem is converted to the optimal regulation of the system (8) and the performance index function (9). Similar to the reference (Liao et al., 2011), we can obtain the following theorem.

Theorem 1: Suppose $(\tilde{A}, \tilde{B})$ is stabilizable and $(Q^{1/2}, \tilde{A})$ is detectable. The optimal preview input of system (8) under the performance index function (9) can be expressed as

$$\dot{u}(t) = -R^{-1}\tilde{B}^T P X(t) - R^{-1}\tilde{B}^T g(t)$$

where P is a positive semi-definite matrix satisfying the algebraic Riccati equation

$$\tilde{A}^T P + P\tilde{A} - P\tilde{B}R^{-1}\tilde{B}^T P + Q = 0 \tag{10}$$

and

$$g(t) = \int_0^{l_d} \left[\exp(\sigma \bar{A}_c^T) P \tilde{W} \dot{d}(t+\sigma) \right] d\sigma + \int_0^{l_r} \left[\exp(\sigma \bar{A}_c^T) P \tilde{B}_m \dot{u}_m(t+\sigma) \right] d\sigma$$

the matrix in this formula

$$\bar{A}_c = \tilde{A} - \tilde{B}R^{-1}\tilde{B}^T P \tag{11}$$

is stable.

Because system (8) has the same form with system (3.4) in reference (Liao et al., 2011), their performance index functions and assumptions about preview are also in the same form. There is no difference just from the mathematical angle, so the derivation is completely similar. It is omitted here.

4. Conditions for the existence of the controller

Theorem 1 requires $(\tilde{A}, \tilde{B})$ is stabilizable and $(Q^{1/2}, \tilde{A})$ is detectable, we discuss the circumstances which original system (1) needs to satisfy so that the two conditions are established. Note that the reference model (2) is a given system, so that the A1 must be satisfied.

Lemma 1: Under the assumption of A1, $(\tilde{A}, \tilde{B})$ is stabilizable if and only if (A, B) is stabilizable and the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank.

Proof: According to the PBH criterion (Chen, 1999), $(\tilde{A}, \tilde{B})$ is stabilizable if and only if for any complex number s satisfying $\mathrm{Re}(s) \geq 0$, the matrix

$$U_c = \begin{bmatrix} -sI & C & -C_m & 0 \\ 0 & A - sI & 0 & B \\ 0 & 0 & A_m - sI & 0 \end{bmatrix}$$

is of full row rank. In the same way, use the PBH criterion, due to A_m is stable we can infer that any complex number s satisfying $\mathrm{Re}(s) \geq 0$, $A_m - sI$ is invertible. Therefore, when $\mathrm{Re}(s) \geq 0$, we have

$$\begin{aligned} rank(U_c) &= rank(A_m - sI) + \mathrm{rank}\begin{bmatrix} -sI & C & 0 \\ 0 & A - sI & B \end{bmatrix} \\ &= n_m + \mathrm{rank}\begin{bmatrix} -sI & C & 0 \\ 0 & A - sI & B \end{bmatrix} \end{aligned}$$

That is to say, matrix U_c is of full row rank if and only if $\begin{bmatrix} -sI & C & 0 \\ 0 & A - sI & B \end{bmatrix}$ is of full row rank. Two cases of this matrix are discussed.

(i) When $s=0$, obviously $\begin{bmatrix} -sI & C & 0 \\ 0 & A-sI & B \end{bmatrix}$ is of full row rank if and only if matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank.

(ii) When $\mathrm{Re(s)} \geq 0$ and $s \neq 0$, sI is reversible, thus $\begin{bmatrix} -sI & C & 0 \\ 0 & A-sI & B \end{bmatrix}$ is of full row rank if and only if matrix $\begin{bmatrix} sI-A & B \end{bmatrix}$ is of full row rank.

Note that, from the discussion of $s=0$ is known that $\begin{bmatrix} sI-A & B \end{bmatrix}\big|_{s=0}$ is also of full row rank, therefore, we get the conclusions to be proved by combining these discussions. This accomplishes the proof of the Lemma 1.

Lemma 2: Under the assumption of A1, $(Q^{1/2}, \tilde{A})$ is detectable if and only if (C, A) is detectable.

Proof: According to the PBH criterion, $(Q^{1/2}, \tilde{A})$ is detectable if and only if for any complex number s satisfying $\mathrm{Re}(s) \geq 0$, the matrix $U_o = \begin{bmatrix} \tilde{A}-sI \\ Q^{1/2} \end{bmatrix}$ is of full column rank. Note that

$$U_o = \begin{bmatrix} -sI & C & -C_m \\ 0 & A-sI & 0 \\ 0 & 0 & A_m - sI \\ Q_e^{1/2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

let s is a complex number and satisfying $\mathrm{Re}(s) \geq 0$, from the structure of matrix U_o and the invertibility of matrix $A_m - sI$ and $Q_e^{1/2}$, we have

$$
\begin{aligned}
rank(U_o) &= rank\begin{bmatrix} -sI & C & -C_m \\ 0 & A-sI & 0 \\ 0 & 0 & A_m - sI \\ Q_e^{1/2} & 0 & 0 \end{bmatrix} \\
&= rank(Q_e^{1/2}) + rank\begin{bmatrix} C & -C_m \\ A-sI & 0 \\ 0 & A_m - sI \end{bmatrix} \\
&= rank(Q_e^{1/2}) + rank(A_m - sI) + rank\begin{bmatrix} A-sI \\ C \end{bmatrix} \\
&= p + n_m + rank\begin{bmatrix} A-sI \\ C \end{bmatrix}
\end{aligned}
$$

From this, and citing the PBH criterion again, it can be proved that Lemma 2 is established.

Theorem 1 is the result of using optimal control theory for augmented system. Lemma 1 and Lemma 2 show the relationship between the stabilization and detectability of augmented system and the corresponding characteristics of the original system. Now we need to go back to the original system to get the controller we required. Using the Lemmas 1 and 2 in Theorem 1 and solving the $u(t)$, we get the final result of this paper.

Theorem 2: If A1, A2, A3 and A4 hold, Q_e and R are positive definite matrices, then, the Riccati equation (10) has a unique symmetric positive semi-definite solution, and the optimal preview input of system (1) under the performance index function (4) can be expressed as

$$
u(t) = u(0) - K_e \int_0^t e(\sigma) d\sigma - K_x[x(t) - x(0)] - K_{x_m}[x_m(t) - x_m(0)] + f_1(t) + f_2(t) \qquad (12)
$$

where $f_1(t)$, $f_2(t)$, K_e, K_x and K_{x_m} are

$$
f_1(t) = -R^{-1}\tilde{B}^T \int_0^{l_d} \exp(\sigma \bar{A}_c^T) P \tilde{W} [d(t+\sigma) - d(\sigma)] d\sigma
$$

$$
f_2(t) = -R^{-1}\tilde{B}^T \int_0^{l_r} \exp(\sigma \bar{A}_c^T) P \tilde{B}_m [u_m(t+\sigma) - u_m(\sigma)] d\sigma
$$

$$
K = R^{-1}\tilde{B}^T P = \begin{bmatrix} K_e & K_x & K_{x_m} \end{bmatrix} \qquad (13)
$$

stability matrix $\bar{A}_c$ is shown as (11). $u(0)$, $x(0)$ and $x_m(0)$ are the initial values that can be arbitrarily taken.

Proof: It can be obtained by utilizing (13)

$$\dot{u}(t) = -K_e e(t) - K_x \dot{x}(t) - K_{x_m} \dot{x}_m(t) - R^{-1}\tilde{B}^T g(t)$$

integrating on $[0,t]$ in both sides, we get

$$u(t) = u(0) - K_e \int_0^t e(\sigma)d\sigma - K_x[x(t) - x(0)] - K_{x_m}[x_m(t) - x_m(0)] - R^{-1}\tilde{B}^T \int_0^t g(s)ds \quad (14)$$

Due to calculation

$$\begin{aligned}\int_0^t g(s)ds &= \int_0^t \int_0^{l_d} \left[\exp(\sigma \bar{A}_c^T) P \tilde{W} \dot{d}(s+\sigma)\right] d\sigma ds + \int_0^t \int_0^{l_r} \left[\exp(\sigma \bar{A}_c^T) P \tilde{B}_m \dot{u}_m(s+\sigma)\right] d\sigma ds \\ &= \int_0^{l_d} \exp(\sigma \bar{A}_c^T) P \tilde{W} \left[\int_0^t \dot{d}(s+\sigma)ds\right] d\sigma + \int_0^{l_r} \exp(\sigma \bar{A}_c^T) P \tilde{B}_m \left[\int_0^t \dot{u}_m(s+\sigma)ds\right] d\sigma \\ &= \int_0^{l_d} \exp(\sigma \bar{A}_c^T) P \tilde{W} [d(t+\sigma) - d(\sigma)] d\sigma + \int_0^{l_r} \exp(\sigma \bar{A}_c^T) P \tilde{B}_m [u_m(t+\sigma) - u_m(\sigma)] d\sigma\end{aligned}$$

substituting this equation into (14) yields the conclusion to be proved.

Notice that the control effect can be improved by selecting the initial values $x(0)$, $x_m(0)$ and $u(0)$.

Remark 4.1: In (12), $-K_e \int_0^t e(\sigma)d\sigma$ is the integral of the output tracking error, that is, the integrator; $-K_x x(t)$ is the state feedback of the fault model (1); $-K_{x_m} x_m(t)$ is the effect of the fault free reference model (2) on the closed loop system; $f_1(t)$ and $f_2(t)$ are the preview compensation of the fault signal and reference input signal, respectively.

The controller given by (12) is called fault-tolerant preview controller.

5. Numerical simulation

Example: The dynamic equation of a real steam generator water level control system (Irving, Miossec, & Tassart, 1980) is

$$y(\mathrm{s}) = \frac{G_1}{s}\left(q_e(\mathrm{s}) - q_v(\mathrm{s})\right) - \frac{G_2}{1+\tau_2 s}\left(q_e(\mathrm{s}) - q_v(\mathrm{s})\right) + \frac{G_3 s}{\tau_1^{-2} + 4\pi^2 T^{-2} + 2\tau_1^{-1} s + s^2} q_e(\mathrm{s})$$

The corresponding state space equation can be descripted by (1), where

$$A=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & a_{22} & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & a_{43} & a_{44}\end{bmatrix},\ B=\begin{bmatrix}b_1\\ b_2\\ 0\\ 1\end{bmatrix},W=\begin{bmatrix}w_1\\ w_2\\ 0\\ 0\end{bmatrix},\ C=\begin{bmatrix}1 & 1 & 0 & G_3\end{bmatrix},$$

$$\begin{cases}a_{22}=-\tau_2^{-1},\ a_{43}=-(\tau_1^{-2}+4\pi^2T^{-2}),\ a_{44}=-2\tau_1^{-1}\\ b_1=G_1, \quad b_2=-G_2\tau_2^{-1}\\ w_1=-G_1, \quad w_2=G_2\tau_2^{-1}\end{cases}.$$

By selecting a set of parameters with a power of 5%, the coefficient matrices in (15) is obtained

$$A=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & -0.02066 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -0.00333 & -0.04773\end{bmatrix},\ B=\begin{bmatrix}0.058\\ -0.19897\\ 0\\ 1\end{bmatrix},\ W=\begin{bmatrix}-0.058\\ 0.19897\\ 0\\ 0\end{bmatrix},$$

$$C=\begin{bmatrix}1 & 1 & 0 & 0.181\end{bmatrix}.$$

In this system, the input vector $u(t)$ is the feedwater flow of the steam generator, and the disturbance vector $d(t)$ is a known fault signal. The purpose of water level control is to eliminate the effects of fault signal $d(t)$ on the water level $y(t)$ of the steam generator by adjusting the flow rate of feedwater $u(t)$. In other words, when the fault occurs, the water level $y(t)$ of the steam generator can be kept at the ideal water level by the design of the controller $u(t)$.

Suppose in the reference model (2)

$$A_m=\begin{bmatrix}-2 & 0.2\\ 0 & -0.7\end{bmatrix},\ B_m=\begin{bmatrix}1\\ 0\end{bmatrix},\ C_m=[1 \quad 1]$$

and the reference input is a step signal

$$u_m(\mathrm{t})=\begin{cases}0, & 0\le t\le 75\\ 10, & t>75\end{cases}$$

Note that the output vector $y_m(t)$ of (2) is the ideal water level of the steam generator.

Then, suppose the fault signal is

$$d(t)=\begin{cases}0, & 0\le t\le 350\\ 2, & t>350\end{cases}$$

The weight matrices of the performance index function (4) are taken as $Q_e=0.009$, $R=5000$.

It is easy to verify that the conditions of theorem 2 are all satisfied. The solution of the Riccati equation and the gain matrix in the fault-tolerant preview controller (12) are

$$P=\begin{bmatrix} 3.170643 & 558.4987 & 131.4901 & -0.55135 & 0.477859 & -1.58307 & -4.96159\\ 558.4987 & 128707 & 25654.74 & -101.53 & 2.77573 & -279.248 & -877.611\\ 131.4901 & 25654.74 & 5810.079 & -23.8474 & 6.491519 & -65.7439 & -206.598\\ -0.55135 & -101.53 & -23.8474 & 0.100913 & -0.04224 & 0.275675 & 0.866385\\ 0.477859 & 2.77573 & 6.491519 & -0.04224 & 0.926958 & -0.23873 & -0.74585\\ -1.58307 & -279.248 & -65.7439 & 0.275675 & -0.23873 & 0.790973 & 2.479922\\ -4.96159 & -877.611 & -206.598 & 0.866385 & -0.74585 & 2.479922 & 7.780668 \end{bmatrix}$$

$K_e=0.0013416$,

$K_x=\begin{bmatrix}0.47265 & 0.067687 & -0.00023721 & -0.000040733\end{bmatrix}$,

$K_{x_m}=\begin{bmatrix}-0.00067081 & -0.0021081\end{bmatrix}$,

where P is positive definite.

The initial conditions are as follows:

$$u(0)=0.1,\quad x(0)=\begin{bmatrix}0.75\\0\\-0.75\\0.3\end{bmatrix},\quad x_m(0)=\begin{bmatrix}0\\0\end{bmatrix}.$$

Figure 3 shows the output response of the water level control system when the preview length of the fault signal $d(t)$ is $l_d=0$ (i.e., without fault signal preview). Figure 4 and Figure5 show the corresponding tracking error and input signal, respectively.

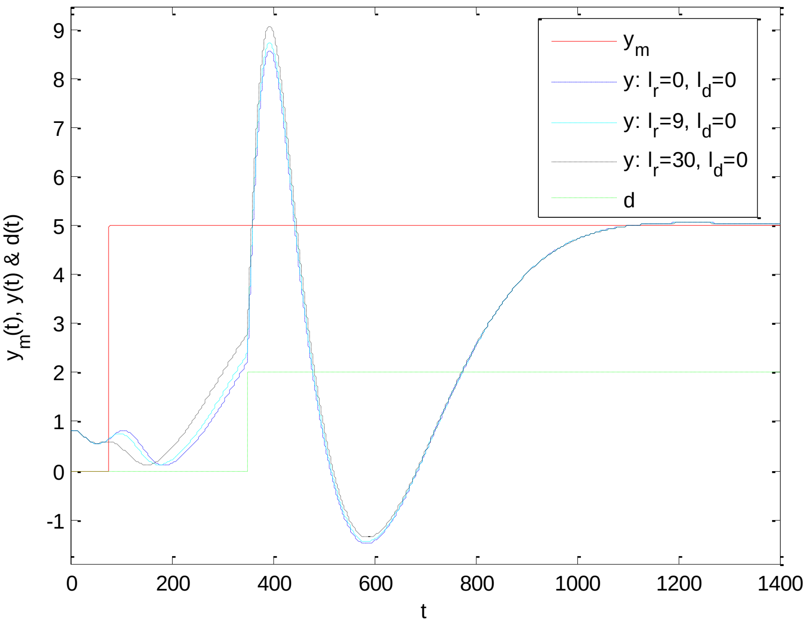

Figure 3. The output response of the water level control system without fault signal preview

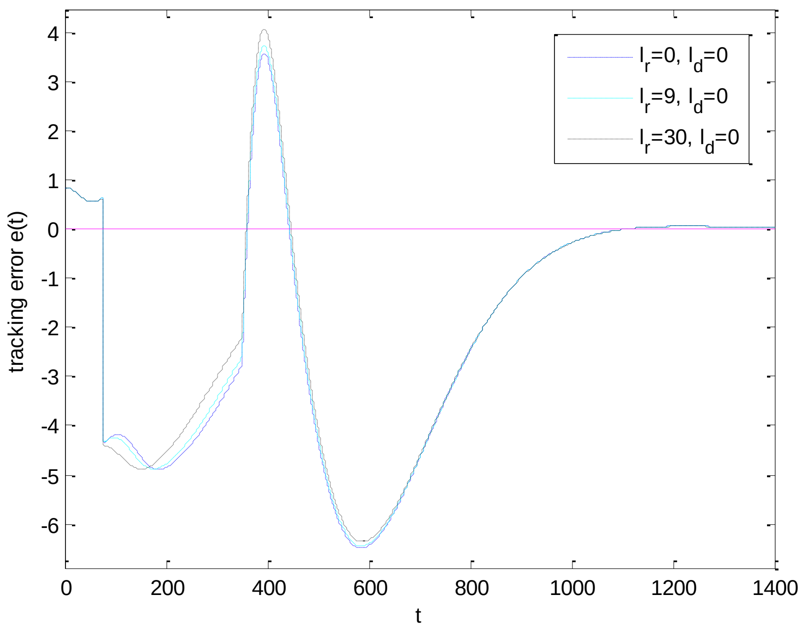

Figure 4. The output error of the water level control system without fault signal preview

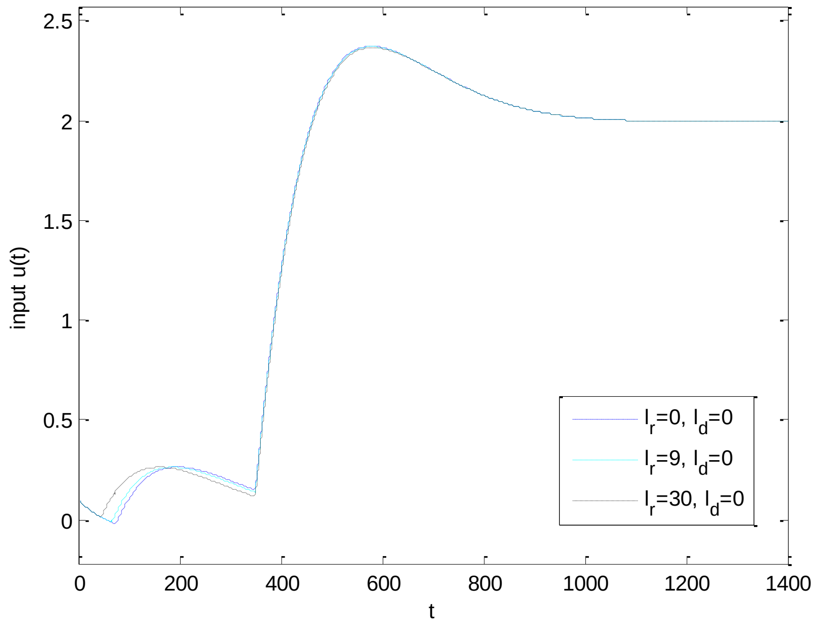

Figure 5. The input of the water level control system without fault signal preview

From Figure 3 and Figure 4, it can be seen that the existence of the preview compensation accelerates the response speed of the system back to the ideal value. In fact, after calculation, we can know that beginning around $t = 450$, the tracking error is largest when $l_r = 0$, the tracking error is smaller when $l_r = 9$, and it is smallest when $l_r = 30$ which indicates the effect of the effect about $u_m(t)$ preview. In the same way, Figure 3 and Figure 4 show that the increase of the preview length of $u_m(t)$ may increase the overshoot in short time when the fault signal is added. These characteristics need to be paid attention to when they are applied, and a proper preview length should be taken. Figure 5 illustrates that once the system overcomes the effects of the fault signal back to the ideal value, the input vector is basically maintained, which is exactly the same as the actual situation.

Figures 6, 7 and 8 reveal the output response, the corresponding tracking error and input signal of the water level control system when the preview length of the reference input signal $u_m(t)$ is $l_r = 0$ (i.e., reference input signal preview), respectively. It can also be noticed that the preview compensation of the fault signal has an effect on the ideal value of the water level recovery. The addition of preview reduces the adverse effects caused by the fault signal, improves the accuracy of the water level tracking ideal value, and the longer the preview length is, the more obvious the effect is. In particular, it is evident from Figure 6 and Figure 7 that the effect of the fault on the change of the water level is effectively suppressed because of the addition of the fault signal preview. Moreover, when the preview length of the fault signal is increased, the adjustment time of the system can be shortened and the overshoot is reduced.

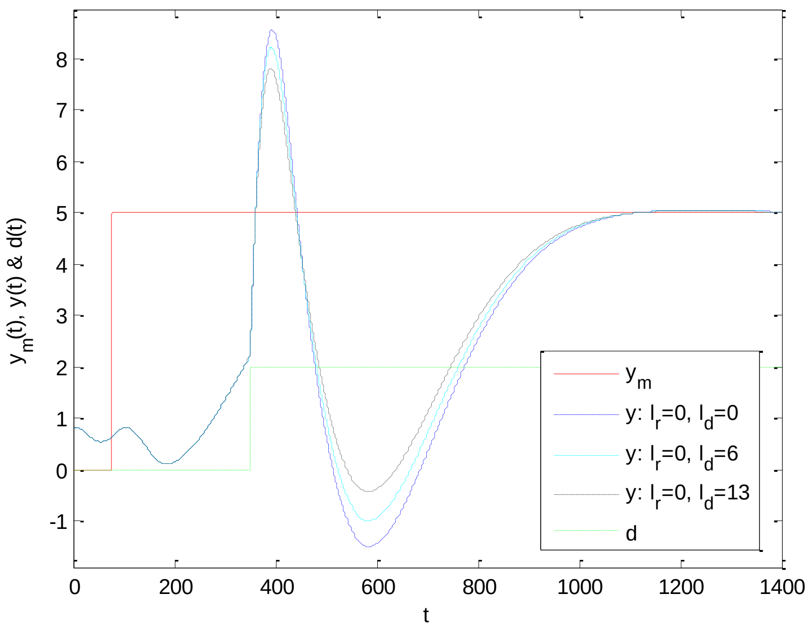

Figure 6. The output response of the water level control system without reference input signal preview

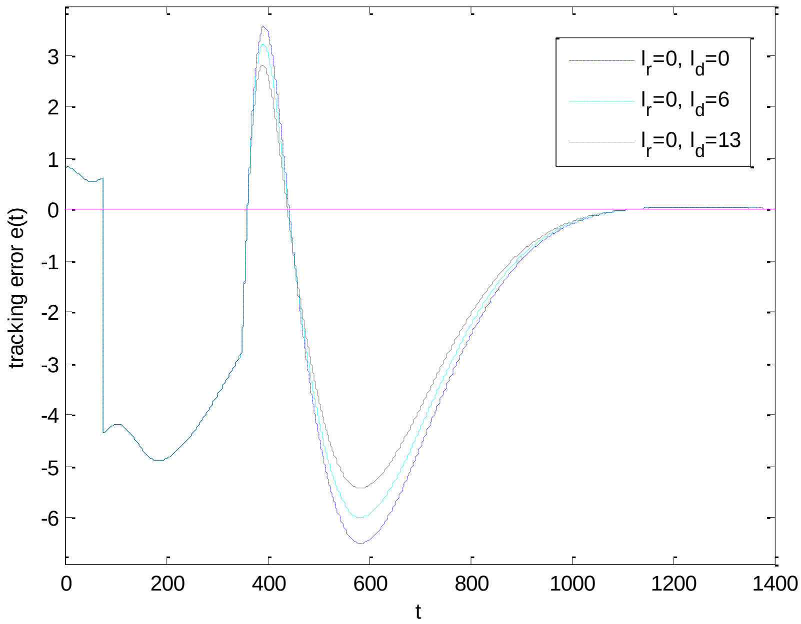

Figure 7. The output error of the water level control system without reference input signal preview

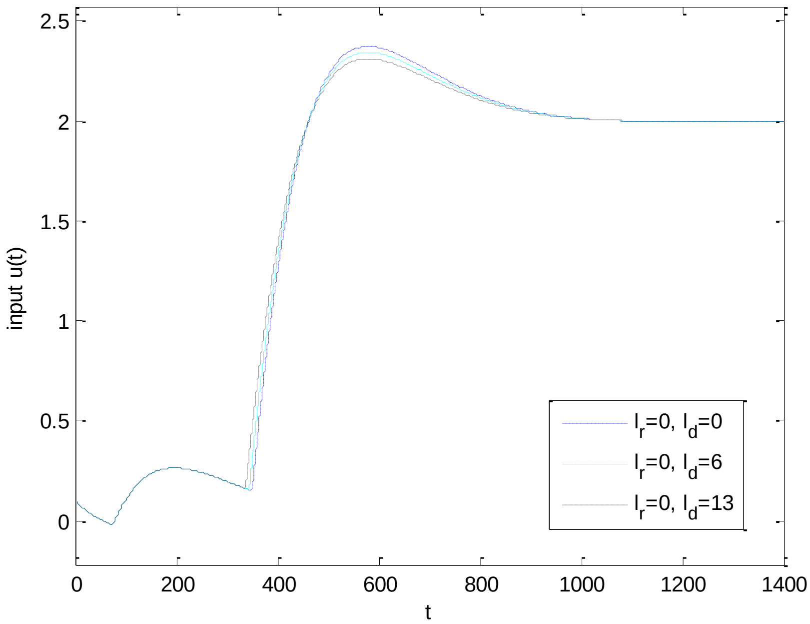

Figure 8. The input of the water level control system without reference input signal preview

Figures 9, 10 and 11 demonstrate the output response, the corresponding tracking error and input signal of the water level control system with the preview of the reference input signal and fault signal, respectively. Similarly, the longer the preview length is, the faster the system will overcome the fault and return to the ideal value.

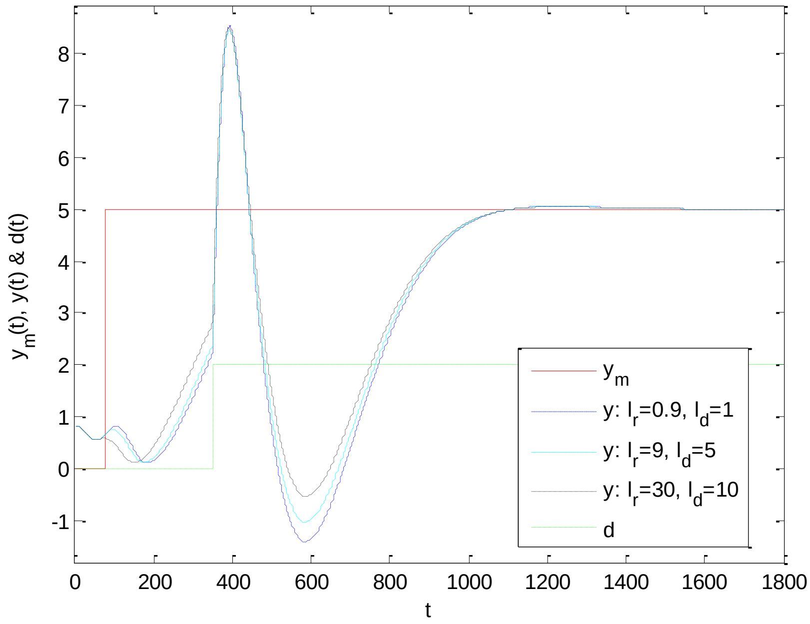

Figure 9. The output response of the water level control system with the preview of the reference input signal and fault signal

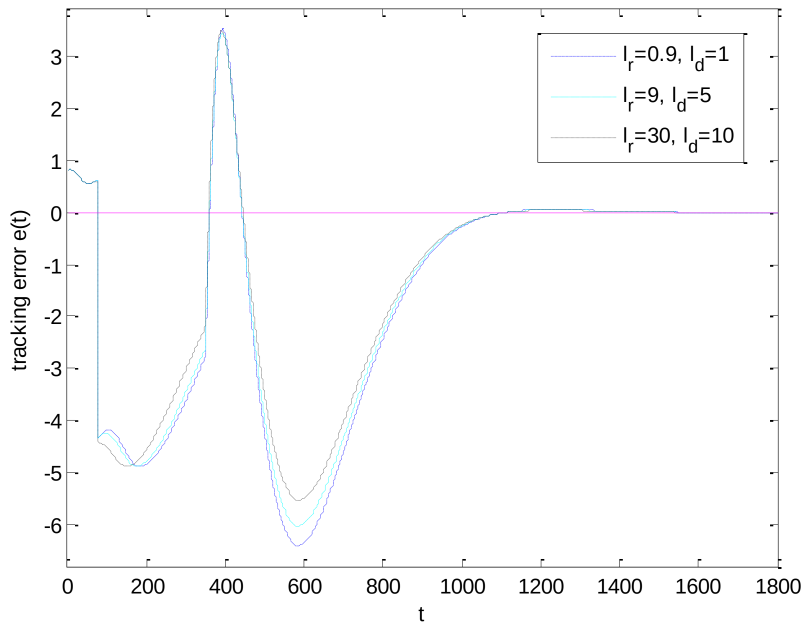

Figure 10. The output error of the water level control system with the preview of the reference input signal and fault signal

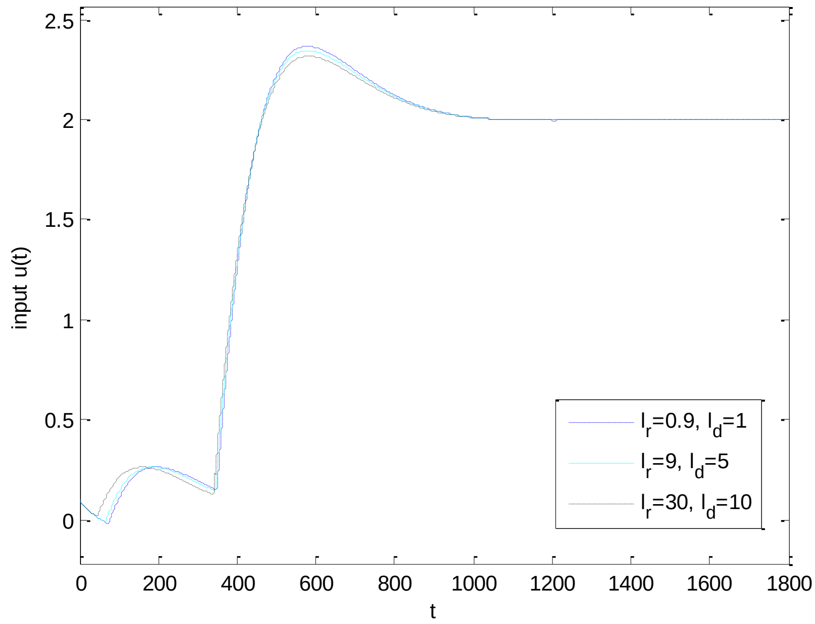

Figure 11. The input of the water level control system with the preview of the reference input signal and fault signal

It should be pointed out that if the fault signal is 0 (i.e., there is no fault signal), the controller designed here is still valid. Figures 12, 13 and 14 show the output response, the corresponding tracking error, and input signal of the water level control system in such condition, respectively. The simulation results show that the effect of water level recovery is noticeable under the reference input preview compensation.

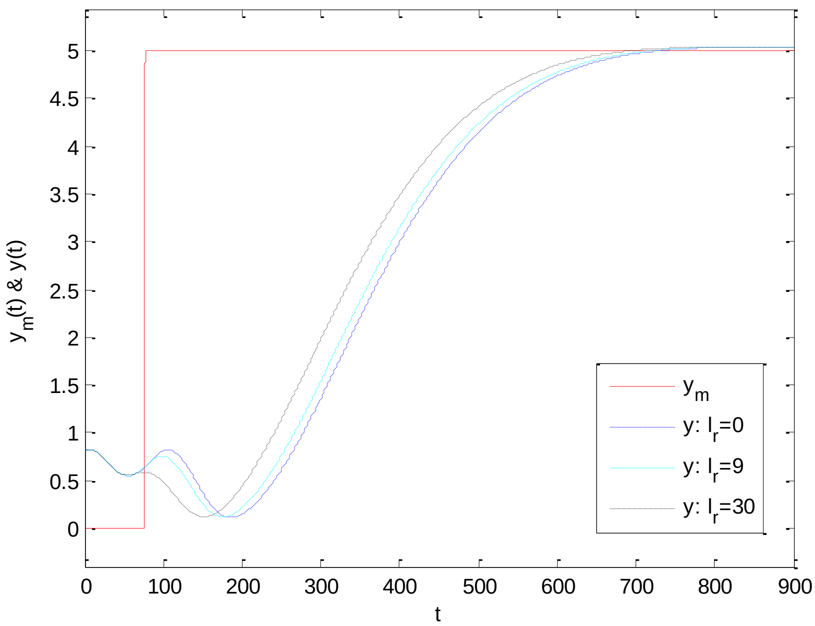

Figure 12. The output response of the water level control system without fault signal

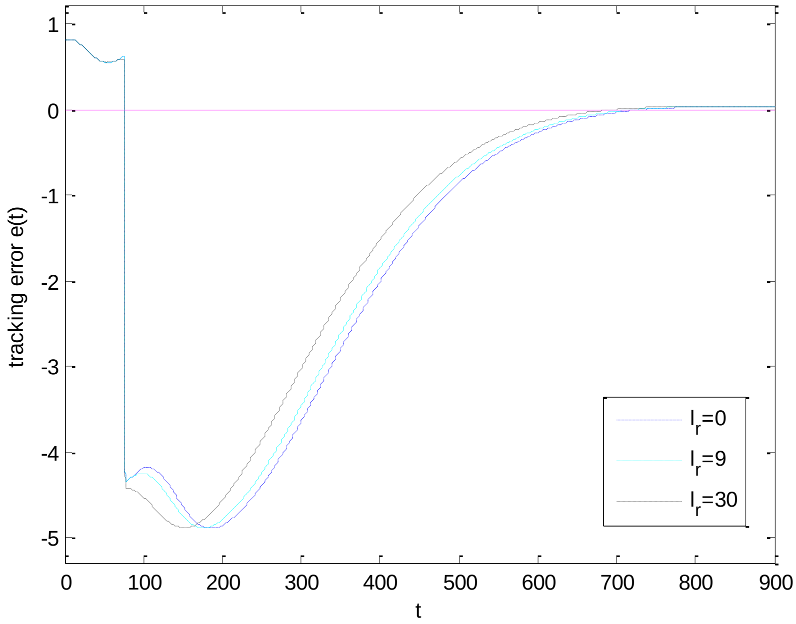

Figure 13. The output error of the water level control system without fault signal

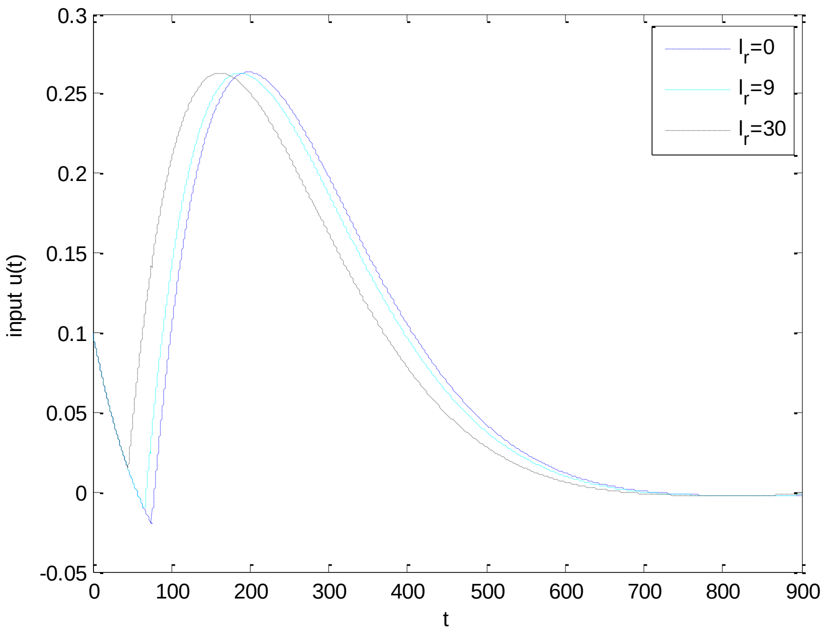

Figure 14. The input of the water level control system without fault signal

Remark 5.1: The controller designed in this paper has two main advantages: firstly, the existence of the preview compensation accelerates the response speed of the system; secondly, the designed control contains integrator, which can eliminate static error(Liao et al. , 2003).

6. Conclusion

In this paper, the theory of the preview control is developed by combining with model-following control, and which is successfully applied to the fault-tolerant problem. The fault-tolerant preview controller for a class of fault systems is designed. Apart from the disturbance preview, it is different from the traditional preview control that the desired signal of the system is the output of the reference model and the previewable signal is the input of the reference model. By constructing augmented system, the problem is transformed into the preview controller design problem of augmented system. Then, the preview controller of the augmented system is obtained by using the similar research method in the preview control problem, from which it can be gained the required fault-tolerant preview controller. Finally, the results are applied to a steam generator water level control system, and the numerical simulation is carried out to illustrate the effectiveness of the proposed method.

In future, one of the important research topics is to extend the present result to the continuous time linear descriptor systems, which is more challenging due to the special structure of descriptor systems. And the issue of the fault-tolerant preview control based on the techniques of sliding mode will be interesting and of great significance.

Funding

This work was supported by National Key R&D Program of China (2017YFF0207401) and the Oriented Award Foundation for Science and Technological Innovation, Inner Mongolia Autonomous Region, China (no. 2012).

Reference

Bodson, M., & Groszkiewicz, J. E. (1997). Multivariable adaptive algorithms for reconfigurable flight control. *IEEE Transactions on Control Systems Technology, 5*(2), 217-229.

Chen, C. T. (1999). Linear system theory and design (Third Edition). *New York: Oxford University Press*.

Hu, S., & Cheng, J. (1991). The model reference fault tolerant control in aircraft. *Acta Aeronautica ET Astronautica Sinica*, *12*(5), A279-A286.

Huang, Q., Qin, J., Liang, A., & Li, H. (2004). Practical technology for steel rolling production. *Metallurgical Industry Press.*

Irving, E., Miossec, C., & Tassart, J. (1980). Toward efficient full automatic operation of the PWR steam generator with water level adaptive control. *Proceedings of the 2nd International Conference on Boiler Dynamics and Control in Nuclear Power Stations*, 309-329.

Katayama, T., & Hirono, T. (1987). Design of an optimal servomechanism with preview action and its dual problem. *International Journal of Control, 45*(2), 407-420.

Kojima, A., & Ishijima, S. (2003). H_∞ performance of preview control systems. *Automatica, 39*(4), 693-701.

Li, L., & Liao, F. (2016). Parameter-dependent preview control with robust tracking performance. *IET control theory and applications, 11*(1), 38-46.

Liao, F., Takaba, K., Katayama, T., & Katsuura, J. (2003). Design of an optimal preview servomechanism for discrete-time systems in a multirate setting. *Dynamics of Continuous Discrete & Impulsive Systems, 5*(5), 727-744.

Liao, F., Tang, Y. Y., Liu, H., & Wang, Y. (2011). Design of an optimal preview controller for continuous-times systems. *International Journal of Wavelets, Multiresolution and Information Processing, 9*(4), 655-673.

Liao, Y., Huang, J., & Zeng, Q. (2010). Preview fuzzy control method for intelligent vehicle path tracking. *Proceedings of IEEE International Conference on Progress in Informatics and Computing, 2*, 1211-1214.

Mizuno, N., Saka, N., & Katayama, T. (2016). A ship's automatic maneuvering system using optimal preview sliding mode controller with adaptation mechanism. *Proceedings of the 10th IFAC Conference on Control Applications in Marine Systems, 49*(23), 576-581.

Nonami, K., & Ikedo, Y. (2004). Walking control of COMET-III using discrete time preview sliding mode controller. *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, 4*, 3219-3225.

Peng, H., & Tomizuka, M. (1993). Preview control for vehicle lateral guidance in highway automation. *Journal of Dynamic Systems, Measurement, and Control, 115*(4), 679-686.

Tao, G., Joshi, S. M., & Ma, X. (2001). Adaptive state feedback and tracking control of systems with actuator failures. *IEEE Transactions on Automatic Control, 46*(1), 78-95.

Tomizuka, M., & Rosenthal, D. E. (1979). On the optimal digital state vector feedback controller with integral and preview actions. *Journal of Dynamic Systems, Measurement, and Control, 101*(2), 172-178.

Tsuchiya, T., & Egami, T. (1994). Digital preview and predictive control. Translated by Liao, F.. *Beijing: Beijing Science and Technology Press*.

Wang, H., Liu, C., & Chen, Q. (2016). Omnidirectional walking based on preview control for biped robots. *Proceedings of IEEE International Conference on Robotics and Biomimetics*, 856-861.

Wang, X., Wang, S., Yang, Z., & Zhang, C. (2015). Active fault-tolerant control strategy of large civil aircraft under elevator failures. *Chinses Journal of Aeronautics, 28*(6), 1658-1666.

Yeh, E. C., & Tsao, Y. J. (1994). Fuzzy preview control scheme of active suspension for rough road. *International Journal of Vehicle Design*, *15*(1-2), 166-180.

Youn, I., Khan, M. A., Uddin, N., & Tomizuka, M. (2017). Road disturbance estimation for the optimal preview control of an active suspension systems based on tracked vehicle model. *International Journal of Automotive Technology, 18*(2), 307-316.

Zhang, Y. M., & Jiang, J. (2002). Active fault-tolerant control system against partial actuator failures. *IEE Proceedings-Control Theory and Applications, 149*(1), 95-104.

Zhao, J., Jiang, B., Chowdhury, F. N., & Shi, P. (2014). Active fault-tolerant control for near space vehicles based on reference model adaptive sliding mode scheme. *International Journal of Adaptive Control and Signal Processing, 28*(9), 765-777.

Notes on contributors

Fucheng Liao received BS degree in Mathematics from Yanan University, Yanan, China, in 1982, MS degree in Mathematics from Northeastern University, Shenyang, China, in 1984, and received his Ph.D. degree in Control Theory and Control Engineering from University of Science and Technology Beijing, Beijing, China, in 2007. Currently, he is a professor in School of Mathematics and Physics, University of Science and Technology Beijing, China. His current research interests are in the area of preview control and robust control.

Chen Jia was born in 1993. She received the Bachelors' degree in Mathematics and Applied Mathematics from Yuncheng University, China, in 2014. She is currently a PhD student at the School of Mathematics and Physics, University of Science and Technology Beijing, China. Her current research interests are in the area of preview control and fault-tolerant control.

Usman Malik received the Masters degree in applied mathematics from international Islamic university Islamabad Pakistan in 2016. He is currently a PhD student at the school of mathematics and physics of the university of science and technology Beijing. His research interests is related to general and fundamental mechanics and preview control.

Xiao Yu was born in 1991. She received the Bachelors' degree in Statistics from Dezhou University, China, in 2014. She is currently a PhD student at the School of Mathematics and Physics, University of Science and Technology Beijing, China. Her current research interest is in the area of preview control.

Professor Jiamei Deng obtained her PhD degree in Reading University on Dec. 2005. She is currently a Professor in Artificial Intelligence and Energy at Leeds Beckett University with a particular interest in control systems design, predictive models, emissions modelling, data-driven power plant safety monitoring.