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Design of preview controller for a type of discrete-time interconnected systems

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Abstract

This article proposes and studies a problem of preview control for a type of discrete-time interconnected systems. First, adopting the technique of decentralized control, isolated subsystems are constructed by splitting the correlations between the systems. Utilizing the difference operator to the system equations and error vectors, error systems are built. Then, the preview controller is designed for the error system of each isolated subsystem. The controllers of error systems of isolated subsystems are aggregated as a controller of the interconnected system. Finally, by employing Lyapunov function method and the properties of non-singular M-matrix, the guarantee conditions for the existence of preview controllers for interconnected systems are given. The numerical simulation shows that the theoretical results are effective.

Keywords

Discrete-time interconnected systems, preview control, error system, Lyapunov function, non-singular M-matrix

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Introduction

In many practical cases, future reference or disturbance signal of control systems is either partly or completely known, such as the flight path of aircraft, the processing path of numerically controlled machine tools, the driving path of vehicles, and so on. This future information is fully utilized to improve the quality of system, which is the preview control problem. Since Sheridan[1] put forward the concept of preview control, which was further upheld by Masayoshi Tomizuka, Tohru Katayama, and other scholars in the 1960s, preview control has attracted extensive attention in theoretical research and application, and formed a set of relatively complete theories and methods.[2–4] Preview control theory has been widely combined with various systems in recent years, which has produced many important results. In Liao et al.,[5] preview control theory and descriptor systems are merged to investigate preview control for linear causal descriptor systems. The theory results of preview control were extended to the cooperative consensus problem of multi-agent systems, and the sufficient conditions to guarantee the achievement of cooperative preview tracking control were given.[6] In addition, the theory of multi-rate systems preview control and random systems preview control has made progress.[7,8] At the same time, preview control has also been exploited in many engineering control problems, such as robot system, active suspension system, electromechanical servo, and aircraft.[9–11]

The so-called interconnected system refers to the system with complex structure, comprehensive functions, numerous factors, and large scale. Interconnected system is also called large-scale system. The power systems, urban transportation networks, and water resources systems are the examples of actual interconnected systems, which can be seen in daily life.[12–14] For interconnected systems, if the controller is designed by centralized control method, it will be difficult to centralize and deal with a large amount of information, which makes the control difficult to achieve. Therefore, decentralized aggregation method is adopted to design the controller[15–17] from a mathematical perspective, that is, the large-scale systems decomposition method. First, the associated terms are deleted artificially to obtain several low-dimensional systems (called isolated subsystems), and controllers are designed to meet

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certain requirements. Then, the controller of the interconnected system is obtained through a certain method of synthesis.[18–20] So far, there have been many control theories for interconnected systems. The tracking problem of interconnected systems through decentralized iterative learning control was studied in the following literature.[21–23] Zhang and Feng[23] reported the problem of controller design for fuzzy interconnected systems, and its stability is analyzed using piecewise Lyapunov function. Furthermore, Koeln[24] discusses the decentralized control of interconnected systems with a special structure, and studies its application in large refrigeration and air conditioning systems.

Under the current circumstances that theories of preview control and interconnected systems have made great progress, it has important theoretical and practical significance to associate with the two theories. Until now, only Liao et al.[25] solved a type of preview tracking control problems related to continuous-time interconnected systems. This article designs a controller with preview effect for a type of discrete-time interconnected systems, taking into account the previewable situation of both the external disturbance signal and the reference signal.

Research contents are arranged as follows: The introduction is given in section "Introduction." Section "Preliminaries" consist of the elementary knowledge, which gives the key concepts needed in this paper. Section "Problem formulation" presents preview control problems for a type of interconnected systems and gives fundamental assumptions. The design of error system controller of isolated subsystem and controller of interconnected systems are discussed in sections "Controller of error system of isolated subsystems" and "Preview controller design for interconnected systems," respectively. Section "Numerical simulation" explains numerical simulation. Finally, a brief conclusion is given in section "Conclusion."

Throughout this paper, $A \in R^{n \times m}$ represents A as the real matrix of $n \times m$; $R > 0 (R \geqslant 0)$ shows the matrix R is a symmetric positive definite (semi-positive definite); $B \cdot C$ indicates the Hadamard product of matrices B and C; $\rho(\cdot)$ is the spectral radius of matrix; Δ denotes the difference operator, which means $\Delta\xi(k) = \xi(k) - \xi(k-1)$; and $\|A\|$ means the norm of matrix A derived from Euclid norm of vector.

Preliminaries

For readability, the definition and partial properties of Hadamard product and non-singular M-matrix are given here.

Definition 1. Set $B = [b_{ij}], C = [c_{ij}] \in R^{m \times n}$. $B \cdot C$ is a matrix obtained by multiplying the corresponding element of B and C, that is, $B \cdot C = [b_{ij} c_{ij}]$. Let us call $B \cdot C$ the Hadamard product of matrices B and C.[26]

The following properties can be obtained instantly from Definition 1 and the definition of matrix multiplication.

Property 1. Seting $a = [a_1 \quad a_2 \quad \dots \quad a_n]^T \in R^n$, $d = [d_1 \; d_2 \; \cdots \; d_n]^T \in R^n$, $z_i \in R (i = 1, 2, \dots, n)$, there is

$$[z_1 \quad z_2 \quad \cdots \quad z_n](a \cdot d) = a^T diag(z_1, z_2, \dots, z_n) d$$

Definition 2. Let $A \in R^{n \times n}$ be defined as

$$A = sI_n - B$$

where $s > 0$, each element in matrix B is non-negative. If $s > \rho(B)$, then A is called a non-singular M-matrix.

Lemma 1. If the non-diagonal elements of matrix A are less than or equal to zero, then the necessary and sufficient condition for A to be a non-singular M-matrix is that one of the following conditions must be true:

1. For any $\alpha \geqslant 0$, $A + \alpha I$ is non-singular and
2. There is a matrix $K = diag(k_1, k_2, \dots k_n) > 0$ that makes $KA + A^T K > 0$.

It can be proved that the non-singular M-matrix also has the following property.

Theorem 1. If A is a non-singular M-matrix, $G = (I - A)(I + A)^{-1}$, then there is diagonal matrix $K > 0$, such that $K - G^T K G > 0$.

Proof. From Lemma 1, there is a diagonal matrix $K > 0$, so that $KA + A^T K > 0$. We know from the obvious equality

$$\begin{aligned} 2(KA + A^T K) &= A^T KA + KA + A^T K + K \\ &\quad - (A^T KA - KA - A^T K + K) \\ &= (I + A)^T K (I + A) \\ &\quad - (I - A)^T K (I - A) \stackrel{def}{=} \Phi \end{aligned}$$

that Φ is a positive definite matrix. Since $(I + A)^{-1}$ exist, there is

$$\left[(I + A)^{-1}\right]^T \Phi (I + A)^{-1} = K - G^T K G$$

which means $K - G^T K G$ and Φ are congruent. Because the congruent matrix has the same positivity, $K - G^T K G > 0$. Theorem 1 is proved.

Problem formulation

Consider discrete-time interconnected system

$$\left\{\begin{array}{l}\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_N(k+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & A_2 & \cdots & A_{2N} \\ \vdots & \vdots & & \vdots \\ A_{N1} & A_{N2} & \cdots & A_N \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_N(k) \end{bmatrix} \\ \quad + \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_N \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_N(k) \end{bmatrix} + \begin{bmatrix} E_1 & 0 & \cdots & 0 \\ 0 & E_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_N \end{bmatrix} \begin{bmatrix} d_1(k) \\ d_2(k) \\ \vdots \\ d_N(k) \end{bmatrix} \\ y(k) = \begin{bmatrix} C_1 & C_2 & \cdots & C_N \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_N(k) \end{bmatrix} \end{array}\right. \tag{1}$$

Clearly, system (1) is able to be equivalently expressed as

$$\left\{\begin{array}{l} x_i(k+1) = A_i x_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij} x_j(k) + B_i u_i(k) + E_i d_i(k), \quad i = 1, 2, \ldots, N \\ y(k) = \sum_{i=1}^{N} y_i(k) = \sum_{i=1}^{N} C_i x_i(k) \end{array}\right. \tag{2}$$

Here, $x_i(k) \in R^{n_i}$, $u_i(k) \in R^{m_i}$, $d_i(k) \in R^{q_i}$, $y_i(k) \in R^p$, $A_i \in R^{n_i \times n_i}$, $A_{ij} \in R^{n_i \times n_j}$, $B_i \in R^{n_i \times m_i}$, $E_i \in R^{n_i \times q_i}$, $C_i \in R^{p \times n_i}$ are constant matrices; $A_{ij}(i \neq j)$ is the correlation matrix.

First, the basic assumptions are given as the following:

Assumption 1. (A_i, B_i) is stabilizable and $rank\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix} = n_i + p \quad (i = 1, 2, \ldots, N)$.

Assumption 2. (C_i, A_i) is observable and A_i is invertible $(i = 1, 2, \ldots, N)$.

Assumption 3. The reference signal $r(k)$ is M_r-steps previewable, that is, at the current time k, the reference signals $r(k)$, $r(k+1)$, $r(k+2)$, …, $r(k+M_r)$ are available. And there is

$$r(k+j) = r(k+M_r)(j > M_r)$$

Assumption 4. The disturbance signal $d_i(k)$ is M_{d_i}-steps previewable, that is, at the current time k, the disturbance signals $d_i(k)$, $d_i(k+1)$, $d_i(k+2)$, …, $d_i(k+M_{d_i})$ are available. And there is

$$\begin{gathered} d_i(k+j) = d_i(k+M_{d_i}) \\ (j > M_{d_i}, i = 1, 2, \ldots, N) \end{gathered}$$

Remark 1. Assumption 1 and *Assumption 2* are fundamental assumptions for the original system. In the design of the controller, it is necessary to build the error system (10) and take the performance index function of system (11). The controller of interconnected system is obtained under the conditions where (Φ_i, G_i) is stabilizable and $(Q_i^{1/2}, \Phi_i)$ is observable. Naturally, it is necessary to give conditions that the original system satisfies, so that (Φ_i, G_i) is stabilizable and $(Q_i^{1/2}, \Phi_i)$ is observable. According to Katayama et al.,[4] *Assumption* 1 can guarantee that (Φ_i, G_i) is stabilizable. *Assumption* 2 can guarantee that $(Q_i^{1/2}, \Phi_i)$ is observable. *Assumption* 3 and *Assumption* 4 are the standard assumptions for preview control. In fact, by the characteristics of the control systems, only a period of previewable information has obvious impact on quality of the systems. The value beyond previewable steps has a little effect on the characteristics of the systems. Therefore, in the theory of preview control, the future value outside the previewable information is usually assumed to be a constant.[2]

The tracking error $e(k)$ can be defined as follows

$$e(k) = r(k) - y(k) \tag{3}$$

The aim of this article is to adopt optimal control theory to design a previewable controller that allows the output $y(k)$ of the system (2) to asymptotically track $r(k)$. In other words, $\lim_{k\to\infty} e(k) = \lim_{k\to\infty} [r(k) - y(k)] = 0$.

Controller of error system of isolated subsystems

For the sake of designing the controller, the method of decentralized control is utilized. First, we cut off the linkage between subsystems to form isolated subsystems and design controller for each isolated subsystem. Then, the controllers of the isolated subsystems are combined to get the controller of the interconnected systems. Finally, by discussing the stability of interconnected system, the constraints of associated terms are obtained.

Based on the output of system (2), we rewrite $e(k)$ as follows

$$e(k) = \sum_{i=1}^{N} e_i(k) = \sum_{i=1}^{N} [\alpha_i r(k) - y_i(k)] \tag{4}$$

where $\alpha_i (i = 1, 2, \ldots, N)$ are constant and satisfy $\sum_{i=1}^{N} \alpha_i = 1$. According to equation (4), if for any $e_i(k) = \alpha_i r(k) - y_i(k) (i = 1, 2, \ldots, N)$, there is $\lim_{k\to\infty} e_i(k) = 0$, then $\lim_{k\to\infty} e(k) = 0$.

Remark 2. We can think of $y_i(k)$ as the output of ith subsystem. Equation (4) means that if output $y_i(k)$ of ith subsystem tracks $\alpha_i r(k) (i = 1, 2, \ldots, N)$, then the output $y(k) = \sum_{i=1}^{N} y_i(k)$ of interconnected system (1) can track $r(k)$. The parameter $\alpha_i (i = 1, 2, \ldots, N)$ gives us the freedom of choice. For example, we can choose $\alpha_1 = \alpha_2 = \cdots = \alpha_N = (1/N)$, which means that all $y_i(k)$ keep track of $(1/N)r(k)$. If $\alpha_i = 0$, it indicates that the output of the ith subsystem tracks the zero vector, and the task of tracking $r(k)$ is completed by the output of other subsystems, and so on.

The equation of ith isolated subsystem is

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u_i(k) + E_i d_i(k) \\ y_i(k) = C_i x_i(k) \end{cases} \tag{5}$$

Currently, the error system is constructed for the isolated subsystem by the method of usually preview control. As a result, the tracking problem of isolated subsystem is turned into the error system regulation problem. Since the error system of interconnected system is still needed in the construction of the controller of the interconnected system, the error system (2) is constructed first to avoid the repetition calculations. Then, the correlation term is cut off to obtain the error systems of the isolated subsystems.

The Δ is applied on both ends of the state equation of system (2) to get

$$\Delta x_i(k+1) = A_i \Delta x_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij} \Delta x_j(k) + B_i \Delta u_i(k) + E_i \Delta d_i(k) \ \ (i = 1, 2, \ldots, N) \tag{6}$$

Utilizing Δ to both sides of $e_i(k+1) = \alpha_i r(k+1) - y_i(k+1) (i = 1, 2, \ldots, N)$, we get

$$\begin{aligned} \Delta e_i(k+1) &= \alpha_i \Delta r(k+1) - \Delta y_i(k+1) \\ &= \alpha_i \Delta r(k+1) - C_i \Delta x_i(k+1) \\ &\quad (i = 1, 2, \ldots, N) \end{aligned} \tag{7}$$

Notice that $\Delta e_i(k+1) = e_i(k+1) - e_i(k)$, then substitute equation (6) into equation (7) to get

$$\begin{aligned} e_i(k+1) &= e_i(k) + \alpha_i \Delta r(k+1) - C_i A_i \Delta x_i(k) \\ &\quad - \sum_{\substack{j=1 \\ j\neq i}}^{N} C_i A_{ij} \Delta x_j(k) - C_i B_i \Delta u_i(k) \\ &\quad - C_i E_i \Delta d_i(k) \ \ (i = 1, 2, \ldots, N) \end{aligned} \tag{8}$$

Combine equations (6) and (8) to get

$$\begin{aligned} X_i(k+1) &= \Phi_i X_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} X_j(k) + G_i \Delta u_i(k) \\ &\quad + G_{r_i} \Delta r(k+1) + G_{d_i} \Delta d_i(k) \\ &\quad (i = 1, 2, \ldots, N) \end{aligned} \tag{9}$$

Here

$$X_i(k) = \begin{bmatrix} e_i(k) \\ \Delta x_i(k) \end{bmatrix}, \ \Phi_i = \begin{bmatrix} I & -C_i A_i \\ 0 & A_i \end{bmatrix},$$
$$\Phi_{ij} = \begin{bmatrix} 0 & -C_i A_{ij} \\ 0 & A_{ij} \end{bmatrix}, \ G_i = \begin{bmatrix} -C_i B_i \\ B_i \end{bmatrix},$$
$$G_{r_i} = \begin{bmatrix} \alpha_i I \\ 0 \end{bmatrix}, \ G_{d_i} = \begin{bmatrix} -C_i E_i \\ E_i \end{bmatrix}$$

System (9) is the error system of interconnected system (2).

Noted that $e_i(k)$ is a partial vector of $X_i(k)$, so if there is $\lim_{k\to\infty} X_i(k) = 0$ in system (9), there is $\lim_{k\to\infty} e_i(k) = 0 (i = 1, 2, \ldots, N)$. In this way, $y(k)$ of the interconnected system (2) can track $r(k)$ asymptotically.

The error system of isolated subsystems is collected by cutting off the correlation item in system (9). The error system of the ith $(i = 1, 2, \ldots, N)$ isolated subsystem is

$$\begin{aligned} X_i(k+1) &= \Phi_i X_i(k) + G_i \Delta u_i(k) \\ &\quad + G_{r_i} \Delta r(k+1) + G_{d_i} \Delta d_i(k) \end{aligned} \tag{10}$$

For the sake of utilizing the results of optimal control, a quadratic performance index function is defined for error system (10)

$$\tilde{J}_i = \sum_{k=1}^{\infty} \left[X_i^T(k) Q_i X_i(k) + \Delta u_i^T(k) H_i \Delta u_i(k) \right] \tag{11}$$

where $Q_i = \begin{bmatrix} Q_{e_i} & 0 \\ 0 & Q_{x_i} \end{bmatrix} \in R^{(p+n_i)\times(p+n_i)}$, $Q_{e_i} > 0$, $Q_{x_i} \geqslant 0$, $H_i \in R^{m_i \times m_i}$, $H_i > 0$.

Remark 3. Obviously, the input $\Delta u_i(k)$ of the system (10) is used in system (11). For original system (2), it is to quote $\Delta u_i(k)$ (not $u_i(k)$) in the performance index function. This causes the controller to include integrators, which helps to eliminate static errors.[2,4]

From the known conclusion in Katayama et al.,[4] Theorem 2 can be proved directly.

Theorem 2. Let us assume that (Φ_i, G_i) is stabilizable, $(Q_i^{1/2}, \Phi_i)$ is observable, and Assumption 3 and Assumption 4 hold, then the controller of the system (10), which minimizes the performance index function of system (11), has the form of

$$\Delta u_i(k) = F_i X_i(k) + \sum_{j=1}^{M_r} F_{r_i}(j)\Delta r(k+j) + \sum_{j=0}^{M_{d_i}} F_{d_i}(j)\Delta d_i(k+j)$$
$$= F_{e_i}e_i(k) + F_{x_i}\Delta x_i(k) + \sum_{j=1}^{M_r} F_{r_i}(j)\Delta r(k+j) + \sum_{j=0}^{M_{d_i}} F_{d_i}(j)\Delta d_i(k+j) \tag{12}$$

where

$$F_i = [F_{e_i} \quad F_{x_i}] = -\left[H_i + G_i^T P_i G_i\right]^{-1} G_i^T P_i \Phi_i,$$
$$F_{r_i}(j) = -\left[H_i + G_i^T P_i G_i\right]^{-1} G_i^T (\xi_i^T)^{j-1} P_i G_{r_i} (j = 1, 2, \ldots, M_r)$$
$$F_{d_i}(j) = -\left[H_i + G_i^T P_i G_i\right]^{-1} G_i^T (\xi_i^T)^{j} P_i G_{d_i} (j = 0, 1, \ldots, M_{d_d})$$
$$\xi_i = \Phi_i + G_i F_i$$

P_i is the positive definite solution of the following Riccati equation

$$P_i = Q_i + \Phi_i^T P_i \Phi_i - \Phi_i^T P_i G_i\left[H_i + G_i^T P_i G_i\right]^{-1} G_i^T P_i \Phi_i \tag{13}$$

Preview controller design for interconnected systems

The vector

$$\Delta u(k) = \begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \\ \vdots \\ \Delta u_N(k) \end{bmatrix} \tag{14}$$

is constructed, where $\Delta u_i (i = 1, 2, \ldots, N)$ is determined by equation (12). $\Delta u(k)$ is utilized as the controller of the error system (9). Let us discuss the conditions under which the correlation term satisfies, so that the state vector $X_i(k)(i = 1, 2, \ldots, N)$ of the closed-loop system (9) tends to zero asymptotically.

By substituting equation (14) into system (9), the closed-loop system

$$X_i(k+1) = \xi_i X_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} X_j(k) + \Theta_i(k) \quad (i = 1, 2, \ldots, N) \tag{15}$$

can be obtained, here

$$\Theta_i(k) = \sum_{j=1}^{M_r} G_i F_{r_i}(j)\Delta r(k+j) + \sum_{j=0}^{M_{d_i}} G_i F_{d_i}(j)\Delta d_i(k+j) + G_{r_i}\Delta r(k+1) + G_{d_i}\Delta d(k) \ (i = 1, 2, \ldots, N)$$

Next, a sufficient condition is given to assure system (15) asymptotically approaches the zero vector.

Theorem 3. Suppose

1. (Φ_i, G_i) is stabilizable and $(Q_i^{1/2}, \Phi_i)$ is observable $(i = 1, 2, \ldots, N)$
2. *Assumption 3* and *Assumption 4* hold
3. Matrix $L = (T - S)(T + S)^{-1}$ is a non-singular M-matrix

then, the state vector $X_i(k)(i = 1, 2, \ldots, N)$ of system (15), that is, the closed-loop system (9) tends to zero asymptotically.

Here

$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NN} \end{bmatrix}, \ T = diag(\gamma_1, \gamma_2, \ldots, \gamma_N),$$

where $s_{ij} = \|\Phi_{ij}\|(i, j = 1, 2, \ldots, N, i \neq j)$, $s_{ii} = \|\xi_i\|$, $\gamma_i = (s_{ii}^2 + \eta_i \upsilon_i^{-1})^{1/2}$, and $\upsilon_i = \|P_i\|$, $\eta_i = -\lambda_{\max}[\xi_i^T P_i \xi_i - P_i]$ $(i, j = 1, 2, \ldots, N, i \neq j)$.

Proof. First, it is proved that the zero solution of the homogeneous system

$$X_i(k+1) = \xi_i X_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} X_j(k) \quad (i = 1, 2, \ldots, N) \tag{16}$$

corresponding to system (15) is asymptotically stable.

From Katayama et al.,[4] if $(\bar{\Phi}_i, \bar{G}_i)$ is stabilizable, $(Q_i^{1/2}, \Phi_i)$ is observable, *Assumption 3* and *Assumption 4* are established, then there is a unique positive definite solution matrix P_i for Riccati equation (13).[28] Using P_i to construct $V_i(X_i) = X_i^T P_i X_i$, it is a positive definite quadratic form of X_i. Take difference to $V_i(X_i)$ along the system (16) trajectory to obtain

$$\Delta V_i|_{(16)} = X_i^T(k+1) P_i X_i(k+1) - X_i^T(k) P_i X_i(k)$$
$$= \left[X_i^T(k)\xi_i^T + \sum_{\substack{j=1 \\ j\neq i}}^{N} X_j^T(k)\Phi_{ij}^T\right] P_i \left[\xi_i X_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} X_j(k)\right] - X_i^T(k) P_i X_i(k)$$
$$= X_i^T(k)\left[\xi_i^T P_i \xi_i - P_i\right] X_i(k) + 2X_i^T(k)\xi_i^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} X_j(k) + \left[\sum_{\substack{j=1 \\ j\neq i}}^{N} X_j^T(k)\Phi_{ij}^T\right] P_i \left[\sum_{\substack{j=1 \\ j\neq i}}^{N} \Phi_{ij} X_j(k)\right]$$

Notice $\eta_i = -\lambda_{\max}[\xi_i^T P_i \xi_i - P_i]$, further to

$$\Delta V_i\big|_{(16)} \leqslant -\eta_i X_i^T(k)X_i(k) + 2X_i^T(k)\xi_i^T P_i \sum_{\substack{j=1\\ j\neq i}}^{N} \Phi_{ij}X_j(k) + \left[\sum_{\substack{j=1\\ j\neq i}}^{N} \Phi_{ij}X_j(k)\right]^T P_i \left[\sum_{\substack{j=1\\ j\neq i}}^{N} \Phi_{ij}X_j(k)\right]$$

Continuously, use the properties of norms to obtain

$$\begin{aligned}
\Delta V_i\big|_{(16)} &\leqslant -\eta_i\|X_i(k)\|^2 + 2\|X_i^T(k)\|\|\xi_i^T\|\|P_i\|\sum_{\substack{j=1\\ j\neq i}}^{N}\|\Phi_{ij}\|\|X_j(k)\| + \left[\sum_{\substack{j=1\\ j\neq i}}^{N}\|\Phi_{ij}\|\|X_j(k)\|\right]^2\|P_i\| \\
&= -\eta_i\|X_i(k)\|^2 + 2s_{ii}v_i\|X_i^T(k)\|\sum_{\substack{j=1\\ j\neq i}}^{N}s_{ij}\|X_j(k)\| + \left[\sum_{\substack{j=1\\ j\neq i}}^{N}s_{ij}\|X_j(k)\|\right]^2 v_i \\
&= -\left[\left(s_{ii}^2v_i+\eta_i\right)^{1/2}\|X_i(k)\|\right]^2 + v_i\left[s_{ii}\|X_i(k)\| + \sum_{\substack{j=1\\ j\neq i}}^{N}s_{ij}\|X_j(k)\|\right]^2 \\
&= \left[\sqrt{v_i}\sum_{j=1}^{N}s_{ij}\|X_j(k)\|\right]^2 - \left[\left(s_{ii}^2v_i+\eta_i\right)^{1/2}\|X_i(k)\|\right]^2 \\
&= \left[\sqrt{v_i}\sum_{j=1}^{N}s_{ij}\|X_j(k)\| - \left(s_{ii}^2v_i+\eta_i\right)^{1/2}\|X_i(k)\|\right]\left[\sqrt{v_i}\sum_{j=1}^{N}s_{ij}\|X_j(k)\| + \left(s_{ii}^2v_i+\eta_i\right)^{1/2}\|X_i(k)\|\right] \\
&= -\left[\sqrt{v_i}\left(\gamma_i\|X_i(k)\| - \sum_{j=1}^{N}s_{ij}\|X_j(k)\|\right)\right]\left[\sqrt{v_i}\left(\gamma_i\|X_i(k)\| + \sum_{j=1}^{N}s_{ij}\|X_j(k)\|\right)\right]
\end{aligned}$$

Let $i = 1, 2, \ldots, N$ to get

$$\begin{bmatrix}\Delta V_1\big|_{(16)}\\ \Delta V_2\big|_{(16)}\\ \vdots\\ \Delta V_N\big|_{(16)}\end{bmatrix} \leqslant -\left[O(T-S)\begin{bmatrix}\|X_1(k)\|\\ \|X_2(k)\|\\ \vdots\\ \|X_N(k)\|\end{bmatrix}\right]\cdot\left[O(T+S)\begin{bmatrix}\|X_1(k)\|\\ \|X_2(k)\|\\ \vdots\\ \|X_N(k)\|\end{bmatrix}\right] \tag{17}$$

where $O = diag(\sqrt{v_1}, \sqrt{v_2}, \ldots, \sqrt{v_N})$.

According to 3, $L = (T-S)(T+S)^{-1}$ is a non-singular M-matrix. From Theorem 1, it is well known that there exists $K = diag(k_1, k_2, \ldots, k_N) > 0$ such that $K - G^T KG > 0$, here

$$G = (I-L)(I+L)^{-1} \tag{18}$$

Substitute L into equation (18) to get

$$\begin{aligned}
G &= [I-(T-S)(T+S)^{-1}][I+(T-S)(T+S)^{-1}]^{-1} \\
&= [(T+S)(T+S)^{-1} - (T-S)(T+S)^{-1}] \\
&\quad [(T+S)(T+S)^{-1} + (T-S)(T+S)^{-1}]^{-1} \\
&= 2S(T+S)^{-1}[2T(T+S)^{-1}]^{-1} \\
&= ST^{-1}
\end{aligned}$$

So, $K - (ST^{-1})^T K(ST^{-1}) > 0$.

Using K and O, the Lyapunov function of system (16) is taken as positive definite quadratic

$$V = \sum_{i=1}^{N}\frac{k_i}{v_i}V_i = \begin{bmatrix}\frac{k_1}{v_1} & \frac{k_2}{v_2} & \cdots & \frac{k_N}{v_N}\end{bmatrix}\begin{bmatrix}V_1\\ V_2\\ \vdots\\ V_N\end{bmatrix} \tag{19}$$

Then, the difference of V along system (16) trajectory is

$$\Delta V\big|_{(16)} = \begin{bmatrix}\frac{k_1}{v_1} & \frac{k_2}{v_2} & \cdots & \frac{k_N}{v_N}\end{bmatrix}\begin{bmatrix}\Delta V_1\big|_{(16)}\\ \Delta V_2\big|_{(16)}\\ \vdots\\ \Delta V_N\big|_{(16)}\end{bmatrix}$$

By substituting system (17) and using Property 1, we get

$$\Delta V\big|_{(16)} \leqslant - \begin{bmatrix} \frac{k_1}{v_1} & \frac{k_2}{v_2} & \cdots & \frac{k_N}{v_N} \end{bmatrix} \left\{ \left[O(T-S) \begin{bmatrix} \|X_1(k)\| \\ \|X_2(k)\| \\ \vdots \\ \|X_N(k)\| \end{bmatrix} \right] \cdot \left[O(T+S) \begin{bmatrix} \|X_1(k)\| \\ \|X_2(k)\| \\ \vdots \\ \|X_N(k)\| \end{bmatrix} \right] \right\}$$

$$= - [\, \|X_1(k)\| \quad \|X_2(k)\| \quad \cdots \quad \|X_N(k)\| \,](T-S)^T O^T diag\left(\frac{k_1}{v_1}, \frac{k_2}{v_2}, \cdots, \frac{k_N}{v_N}\right) O(T+S) \begin{bmatrix} \|X_1(k)\| \\ \|X_2(k)\| \\ \vdots \\ \|X_N(k)\| \end{bmatrix}$$

$$= - [\, \|X_1(k)\| \quad \|X_2(k)\| \quad \cdots \quad \|X_N(k)\| \,](T-S)^T O^T \begin{bmatrix} \frac{k_1}{v_1} & 0 & \cdots & 0 \\ 0 & \frac{k_2}{v_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{k_2}{v_2} \end{bmatrix} O(T+S) \begin{bmatrix} \|X_1(k)\| \\ \|X_2(k)\| \\ \vdots \\ \|X_N(k)\| \end{bmatrix} \tag{20}$$

$$= - [\, \|X_1(k)\| \quad \|X_2(k)\| \quad \cdots \quad \|X_N(k)\| \,]\left[T^T K T - S^T K S\right] \begin{bmatrix} \|X_1(k)\| \\ \|X_2(k)\| \\ \vdots \\ \|X_N(k)\| \end{bmatrix}$$

Transform $T^T KT - S^T KS$ using positive definite diagonal matrix T^{-1} to get

$$(T^{-1})^T[T^T KT - S^T KS]T^{-1} = K - T^{-1}S^T KST^{-1} \\ = K - (ST^{-1})^T K(ST^{-1}) > 0$$

then, $T^T KT - S^T KS > 0$. When λ is utilized to represent the minimum eigenvalue of $T^T KT - S^T KS$, there must be $\lambda > 0$

$$\Delta V\big|_{(16)} \leqslant - \lambda \sum_{i=1}^{N} \|X_i(k)\|^2 = - \lambda \|X(k)\|^2$$

can be proved by substituting equation (20), and $X(k) = [X_1^T(k) \;\; X_2^T(k) \;\; \cdots \;\; X_N^T(k)]^T$ is the state vector of the system (16). Therefore, the difference of V along with system (16) trajectory is negative definite. As a result, the zero solution of system (16) is asymptotically stable.

Then, we prove that $\lim_{k\to\infty} \Theta_i(k) = 0 (i = 1, 2, \ldots, N)$. According to *Assumption 3* and *Assumption 4*, when k tends to infinity, $\Delta r(k)$ and $\Delta d_i(k)$ tend to zero vectors. In view of the relationship between $\Theta_i(k)$ and $\Delta r(k)$, $\Delta d_i(k)$, it is easy to get $\lim_{k\to\infty} \Theta_i(k) = 0 (i = 1, 2, \ldots, N)$. Because system (16) is asymptotically stable and $\lim_{k\to\infty} \Theta_i(k) = 0 (i = 1, 2, \ldots, N)$, according to Theorem 1 of Chen,[29] there is $\lim_{k\to\infty} X_i(k) = 0 (i = 1, 2, \ldots, N)$ in system (15). Hence, Theorem 3 is proved.

Below, we use the relevant parameters of interconnected system (2) to give the conditions, which ensure that (Φ_i, G_i) can be stabilized and $(Q_i{}^{1/2}, \Phi_i)$ can be observed.

According to Katayama et al.,[4] the sufficient and necessary condition for (Φ_i, G_i) to be stabilizable is that $rank \begin{bmatrix} A_i - I_{n_i} & B_i \\ C_i & 0 \end{bmatrix} = n_i + p$ and (A_i, B_i) is stabilizable $(i = 1, 2, \ldots, N)$. If A_i is invertible and (C_i, A_i) is observable, then $(Q_i{}^{1/2}, \Phi_i)$ is observable $(i = 1, 2, \ldots, N)$.

To sum up, one of the main theorems in this paper is as follows.

Theorem 4. Suppose

1. *Assumption 1* to *Assumption 4* hold;
2. $L = (T - S)(T + S)^{-1}$ is a non-singular M-matrix;
3. $Q_{e_i} > 0$, $H_i > 0$ $(i = 1, 2, \ldots, N)$;
4. Let $u_i(k) = 0$, $r(k) = 0$, $x_i(k) = 0$, $d_i(k) = 0 (i = 1, 2, \ldots, N)$ for $k < 0$;

then the controller with preview effect, which enables the output signal of system (2) to track the reference signal asymptotically, is

$$u(k) = \left[u_1^T(k) \quad u_2^T(k) \quad \cdots \quad u_N^T(k) \right]^T \tag{21}$$

where

$$\begin{aligned} u_i(k) = u_i(0) + F_{e_i}\sum_{j=1}^{k} e_i(j) + F_{x_i}[x_i(k) - x_i(0)] \\ + \sum_{j=1}^{M_r} F_{r_i}(j)[r(k+j) - r(j)] \\ + \sum_{j=0}^{M_{d_i}} F_{d_i}(j)[d_i(k+j) - d_i(j)] \\ (i = 1, 2, \cdots, N) \end{aligned} \tag{22}$$

When steps 1–3 of this theorem are true, all the conditions of Theorem 3 are satisfied, so the conclusion of Theorem 3 is true. The controller of system (1) can be achieved by solving $u_i(k)(i = 1, 2, \cdots, N)$ from equation (14) or equation (12).

For a given $i(i = 1, 2, \cdots, N)$

$$\begin{aligned} u_i(s) - u_i(s-1) = F_{e_i}e_i(s) + F_{x_i}[x_i(s) - x_i(s-1)] \\ + \sum_{j=1}^{M_r} F_{r_i}(j)[r(s+j) - r(s-1+j)] \\ + \sum_{j=0}^{M_{d_i}} F_{d_i}(j)[d_i(s+j) - d_i(s-1+j)] \end{aligned} \tag{23}$$

is obtained from equation (12). In equation (23), taking $s = 1, 2, \cdots, k$, adding the two sides and moving $u_i(0)$ to the right side of equation to get equation (22), then combining $u_i(k)(i = 1, 2, \cdots, N)$, the controller of the interconnected system (2) is attained, that is, equation (21).

Remark 4. In equation (22), $F_{x_i}x_i(k)$ is the state feedback, $F_{e_i}\sum_{j=1}^{k} e_i(j)$ is the integrator, $\sum_{j=1}^{M_r} F_{r_i}(j)$ $[r(k+j) - r(j)]$ is the preview feed-forward of reference information, $\sum_{j=0}^{M_{d_i}} F_{d_i}(j)[d_i(k+j) - d_i(j)]$ is the preview feed-forward of disturbance information, $u_i(0)$ is the initial value of input, $F_{x_i}x_i(0)$ is the compensation of initial value.

Numerical simulation

Two examples are given to illustrate the effectiveness of the designed controller in this section.

Example 1. Consider interconnected system with two subsystems (i.e. $N = 2$), $n_1 = 3$, $n_2 = 2$ and the coefficient matrices are

$$A_1 = \begin{bmatrix} 0.5 & 2 & 1.5 \\ 0 & -0.5 & 1 \\ -1.5 & 0 & -5 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0.005 & 0.003 \\ 0.002 & 0.004 \\ 0.002 & -0.005 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad E_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad C_1 = [0.5 \quad 1 \quad 1]$$

$$A_2 = \begin{bmatrix} 1 & 3 \\ 0 & -5 \end{bmatrix} \quad A_{21} = \begin{bmatrix} 0.001 & 0.0015 & -0.001 \\ 0.001 & -0.002 & 0.001 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C_2 = [1 \quad 1]$$

By adopting the Popov-Belevitch-Hautus (PBH) rank criterion, it is known that $(A_i, B_i)(i = 1, 2)$ is controllable and $(A_i, C_i)(i = 1, 2)$ is observable. In addition, it is easy to verify $rank\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix} = n_i + p(i = 1, 2)$, and $A_i(i = 1, 2)$ is an invertible matrix.

Let the weight matrix of the performance index function of system (11) be

$$Q_1 = \begin{bmatrix} Q_{e_1} & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 \\ 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ Q_{e_1} = 30, \ H_1 = 2$$

$$Q_2 = \begin{bmatrix} Q_{e_2} & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ Q_{e_2} = 20, \ H_2 = 3$$

The solution of Riccati equation of two isolated subsystems and the feedback gain matrix of the controller are calculated using MATLAB

$$P_1 = \begin{bmatrix} 76.2583625100376 & -7.1354483853848 & -10.9509238062926 & -615934591770557 \\ -7.1354483853848 & 44.0598854799177 & 31.1938011523749 & 60.3567253364750 \\ -10.9509238062926 & 31.1938011523749 & 171.4334406413428 & 66.2669915269203 \\ -61.5934591770557 & 60.3567253364750 & 66.2669915269203 & 431.9922105508578 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 51.0269298948671 & -24.5802244149587 & -71.2879879528538 \\ -0.245802244149587 & 87.1434148884782 & 165.1725946429253 \\ -71.2879879528538 & 165.1725946429253 & 551.4440506041630 \end{bmatrix}$$

$$F_{e_1} = 0.217624980452534$$

$$F_{x_1} = [1.387583158168247 \quad -0.260959136238370 \quad 4.321642830283796]$$

$$F_{e_2} = 0.163512352801475, \quad F_{x_2} = [-0.417176668167282 \quad 3.728417778359146]$$

Besides

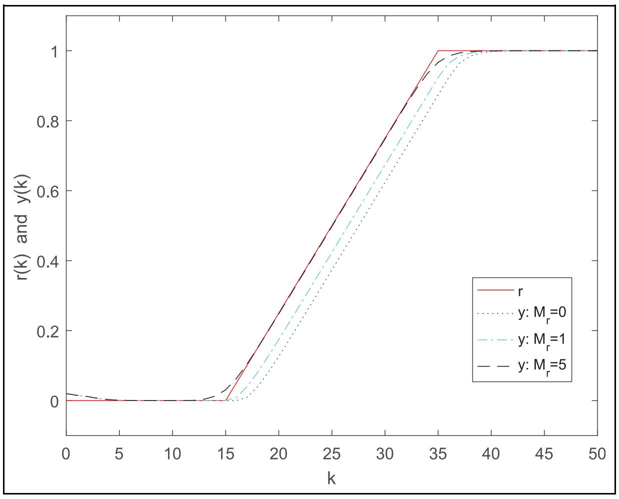

Figure 1. The output response of the interconnected systems without disturbance.

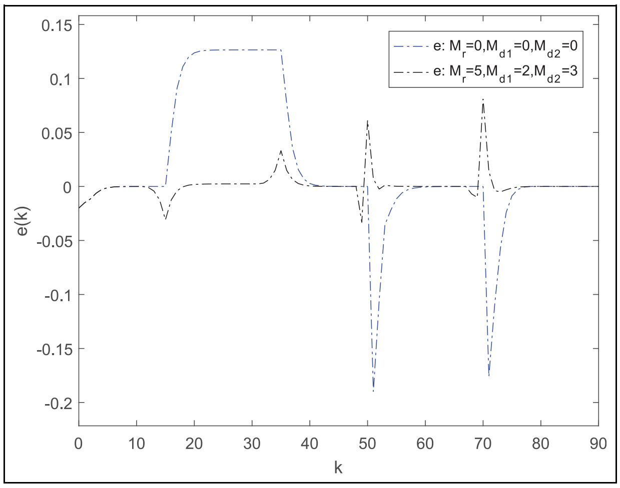

Figure 3. The tracking error of the interconnected system with disturbance.

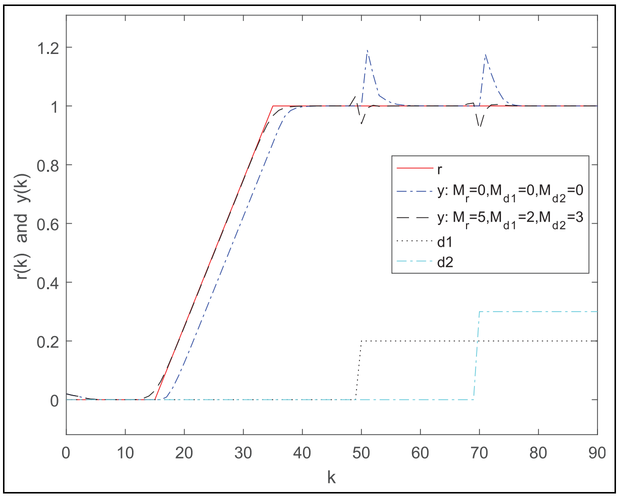

Figure 2. Closed-loop responses of the interconnected system with disturbance.

$$L = \begin{bmatrix} 0.001558426283261 & -0.001166519362468 \\ -0.000531636745513 & 0.000529082708424 \end{bmatrix}$$

The eigenvalues of L are 0.001984526309942 and 0.000102982681743, which means that L is a non-singular M-matrix.

First, considering the non-disturbance situation, that is, $d_i(k) = 0 (i = 1, 2)$, $r(k)$ can be chosen as

$$r(k) = \begin{cases} 0, & k \leqslant 15 \\ 0.05(k-15), & 15 < k \leqslant 35 \\ 1, & k > 35 \end{cases} \tag{24}$$

Let the initial state be $x_1(0) = [0 \quad 0.01 \quad 0]^T$, $u_1(0) = 0$, $x_2(0) = [0.01 \quad 0]^T$, $u_2(0) = 0$. Selecting $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, the three cases of $M_r = 0$, $M_r = 1$, and $M_r = 5$ are numerically simulated.

The tracking effect of interconnected system (2) is shown in Figure 1. We can see that the output of interconnected system (2) is able to track $r(k)$ asymptotically with the increase of time k. Thus, it is clear that the tracking speed can be accelerated using a larger number of preview steps.

Now, consider the situation with disturbance signals as follow

$$d_1(k) = \begin{cases} 0, & k < 50 \\ 0.2, & k \geqslant 50 \end{cases}, \quad d_2(k) = \begin{cases} 0, & k < 70 \\ 0.3, & k \geqslant 70 \end{cases}$$

The reference signal is still in the form of equation (24). At this time, the output curve of the interconnected system is depicted in Figure 2.

The tracking error of interconnected system is depicted by Figure 3. It can be clearly seen from Figures 2 and 3 that the output of interconnected system is able to track the reference signal asymptotically, even if there are disturbance signals. Moreover, the controller with preview effect can apparently decrease the tracking error and the overshoot caused by disturbance.

Example 2. Consider interconnected system (2), where $N = 2$, $n_1 = n_2 = 2$

$$A_1 = \begin{bmatrix} 0.1 & 1 \\ 0 & 0.2 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0.1 & 0.05 \\ 0.02 & 0.1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.2 \\ 0 \end{bmatrix} \quad E_1 = \begin{bmatrix} -1.5 \\ 0 \end{bmatrix} \quad C_1 = [-3 \quad 1]$$

$$A_2 = \begin{bmatrix} 0.1 & -1 \\ 0 & -0.3 \end{bmatrix} \quad A_{21} = \begin{bmatrix} 0.02 & 0.03 \\ 0.05 & 0.1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix} \quad E_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad C_2 = [1.5 \quad 0]$$

After verification, $(A_i, B_i)(i = 1, 2)$ can be stabilized (but not controllable), $(A_i, C_i)(i = 1, 2)$ can be observed, and the matrix $\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix} (i = 1, 2)$ is of full row rank. Let

$$Q_{x_1} = \begin{bmatrix} 20 & 0 \\ 0 & 0 \end{bmatrix}, Q_{e_1} = 30, H_1 = 2$$

$$Q_{x_2} = \begin{bmatrix} 20 & 0 \\ 0 & 0 \end{bmatrix}, Q_{e_2} = 30, H_2 = 3$$

Similarly, the solution of Riccati equation for two isolated subsystems and the feedback gain matrix of the controller are obtained

$$P_2 = \begin{bmatrix} 36.304268573061421 & 1.360259638323270 & 12.574782360793279 \\ 1.360259638323270 & 20.438323123878305 & 4.214645954245819 \\ 12.574782360793279 & 4.214645954245819 & 42.131239677202679 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 52.575702989830198 & -2.756669712831776 & 24.738103635858831 \\ -2.756669712831776 & 20.496692403145200 & -4.544892821345821 \\ 24.738103635858831 & -4.544892821345821 & 44.533491801009028 \end{bmatrix}$$

$$F_{e_1} = 1.360259638323257, F_{x_1} = [0.438323123878289 \quad 4.214645954245810]$$

$$F_{e_2} = 1.837779808554515, F_{x_2} = [-0.331128268763466 \quad 3.029928547563879]$$

By calculation, there is

$$L = \begin{bmatrix} 0.337474647269200 & -0.237820233607235 \\ -0.129901485275353 & 0.093322541770793 \end{bmatrix}$$

L is a non-singular M-matrix.

Reference signal and disturbance signals are selected as

Selecting $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, the initial value is set as $x_1(0) = [0 \quad 0.02]^T$, $u_1(0) = 0$, $x_2(0) = [0.01 \quad 0]^T$, $u_2(0) = 0$. We carried out numerical simulations for two cases $M_r = 0$, $M_{d_1} = 0$, $M_{d_2} = 0$, and $M_r = 6$, $M_{d_1} = 4$, $M_{d_2} = 3$.

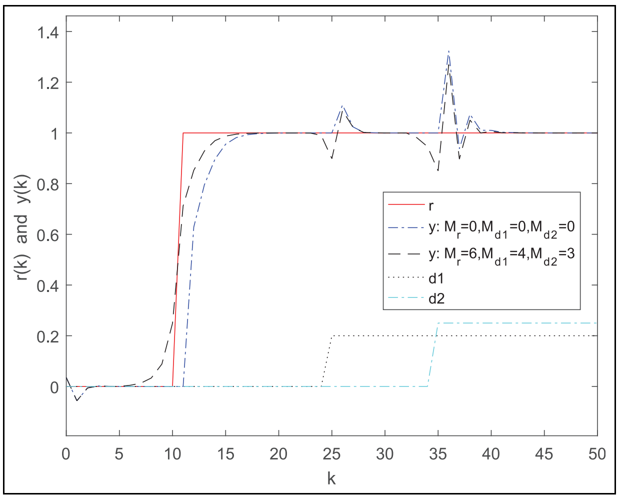

Figure 4. Step response of interconnected system in Example 2.

Figure 4 indicated that the $r(k)$ can be tracked by the $y(k)$ of the interconnected system without static error, and the controller with preview effect can shorten the adjustment time and restrain the disturbance signal to a certain extent.

Conclusion

This paper investigates the previewable controller for a type of discrete-time interconnected systems. Initially, using the basic scheme of preview control, the previewable controller is designed for the error system of each isolated subsystem. Then, the controller of the error system of the isolated subsystem is combined as the controller of error interconnected system. In resort of Lyapunov function and the properties of non-singular M-matrix, the stability of error interconnected system is discussed, then the criterion to ensure its stability is given. Finally, the guarantee conditions for the existence of the preview controller, then the controller for the original interconnected system are derived. The theoretical results and numerical simulation show that the designed controller is able to make the output of the system to track reference signal without static error regardless of the existence of the disturbance signal, and the tracking performance is improved with the increase of the preview steps.

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