Paradox of Stationarity? A policy target dilemma for policymakers

Muhammad Ali Nasir*[1], Jamie Morgan

Leeds, Business School, Leeds Beckett University, United Kingdom.

Abstract: This brief paper sets out an underappreciated practically oriented paradox and its methodological implications. The paradox is highly relevant to central banks and any organization or agent that seeks to match or anticipate central bank policy, such as other actors in banking and finance. Specifically, there is a divergence between the statistical requirement of stationarity and any macroeconomic policy objective that involves a target that takes a consistent and positive value. This is not merely an esoteric issue of interest to statisticians. It has fundamental implications for policy contexts. When achieved, any policy target, such as inflation targeting, will necessarily result in a unit root. As such, an unintended consequence of successful policy is non-stationarity, which means policy is permanently seeking to actualise conditions that are at cross-purposes with typical analytical statistical requirements. The point and its significance are illustrated beginning with a simple AR model and artificial inflation dataset.

Keywords: unit roots, stationarity, macroeconomic policy targets, Central Banking, epistemology-ontology

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[1] Corresponding author: School of Accounting, Finance and Economics, Faculty of Business & Law, Leeds Beckett University, The Rose Bowl, Portland Crescent, Leeds, LS1 3HB, UK, **Email:** m.a.nasir@leedsbeckett.ac.uk, **Telephone:** 0044(0)7508457981 .

1. Introduction

Statistical analysis creates fundamental philosophical issues (e.g. Rieger, 2002; Reiss, 2015) and these are sometimes obscured by practice in the social sciences. Stationarity is one such issue. Data for statistical analysis is often required to be 'stationary' in order for the use of a statistical procedure to be appropriate or warrantable. For example, panel and time series data require stationarity. A variety of published studies report the presence or absence of a unit root in time series or panel data. In statistical analysis a unit root is an undesirable property. It has long been recognized in the social sciences, and in economics in particular, that in the absence of stationarity a unit root as a feature of the data may result in a 'spurious' (ill-constructed and so essentially meaningless) empirical model (see Granger and Newbold, 1974). Further, the absence of stationarity creates problems for the interpretation of data and hence for policy. It is with problems of stationarity and unit roots that we are concerned in this short paper.

Our contention is that unit root problems are *likely* in macroeconomic policy contexts *because* the point of policy is typically to achieve some objective, which introduces a given positive value into the series. The most familiar of these is an inflation target.[2] For example, the Bank of England has an inflation target of 2%, with a 1% range. Beyond the range the Governor of the Bank must justify the deviation to the Chancellor of the Exchequer. Our point is that if the inflation target is achieved (an outcome that can be generalised to, 'if the desired outcome of policy is achieved') then achieving the target will *necessarily* result in a non-mean reverting data series. Thus, successful policy creates an outcome where data has a unit root and is non-stationary. In reality this is rarely and typically only briefly the case, since inflation targets are rarely achieved (rather than approximated). However, policy is continually pulling towards a unit root situation and so this perennial possibility creates a basic problem for the consistency and compatibility of statistical analysis and economic policy. This, in turn, implies problems of epistemology in terms of how statistical analysis is justified, as well as problems of the relation between method and the ontological status of that which is analysed. This notwithstanding,

[2] Given economic growth targets also produce equivalent issues. However, following Meade (1978; see also Tobin, 1980 and Bean, 1983), one can also orient on a policy objective of 'internal balance' with a focus on the growth rate of the economy (GDP).

since much of contemporary statistical analysis is typically predicated on stationarity, data is often transformed to *achieve* stationarity in the data. For example, by differencing. However, one might argue, with the issue of targeting mind, that achieving stationarity through data conformation leads to a problematic point of departure for the empirical tools applied to the analysis of an economic policy objective. Following Rose's terminology we refer to this as a 'paradox of stationarity' (1986). The problems and philosophical implications are easily demonstrated.

2. Producing a unit root

Consider a simple AR (1) model:

$$Y_t = \emptyset Y_{t-1} + e_t \qquad (1)$$

Where e_t is a random noise process. The stationarity condition for the above process is that $|\emptyset|<1$. However, there are three possible cases:

$|\emptyset|<1$ implying the series is stationary.

$|\emptyset|>1$where the series explodes.

$|\emptyset|=1$ where the series contains a unit root and is non-stationary.

As noted in the introduction, a common practise for converting an undesirable non-stationary series to stationarity is by differencing. If we subtract Y_{t-1} from both sides of equation (1):

$$Y_t - Y_{t-1} = Y_{t-1} - Y_{t-1} + e_t \qquad (2)$$

$$\Delta Y_t = e_t$$

e_t is a white-noise process. In this case, the series Y_t contains a unit-root, but when integrated of order one i.e. $Y_t \sim I\ (1)$ then ΔY_t is stationary[3].

Here, there can be a fundamental problem of cross-purpose in a macroeconomic context (our paradox). Modern central banking is focused on price stability and typically involves targets. Targeted inflation

[3] Similarly, a series Y_t will be integrated of order *d (*denoted by $Y_t \sim I\ (d)$) if Y_t is non-stationary, but $\Delta^d Y_t$ *is stationary* (See Asteriou & Hall, 2016).

takes a positive value. Consider again Equation (1): $Y_t = \emptyset Y_{t-1} + e_t$ If we specify the model as inflation and the inflation target is set at 2% and this is *consistently achieved* then Y_t will become equal to Y_{t-1} and $\emptyset$ will become 1 and e_t zero. Since ($\emptyset$) becomes 1 there is a unit root. If we take the difference to remove the unit root then the net of the difference would be zero.

In order to clarify the point, consider Table 1; an artificial inflation dataset in country X for the full year 2015, with a target rate of 2% and where that target is consistently achieved:

Table 1: Monthly Data on Consumer Price Index: 2015

Year	**CPI(Y_t)**	ΔYt	**Year**	**CPI (Y_t)**	ΔY_t
January	2	-	**July**	2	0
February	2	0	**August**	2	0
March	2	0	**September**	2	0
April	2	0	**October**	2	0
May	2	0	**November**	2	0
June	2	0	**December**	2	0

Source: Artificial Data

To establish that the original data was *not* stationary and that its first difference was stationary we can apply a unit root test. In this case, we apply the Augmented Dickey and Fuller (ADF) test. Three alternative regression equations can be used to test for the presence of a unit root.

The first has no constant:

$$\Delta Y_{t-1} = \emptyset Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i+} e_t$$

The second contains a constant but no trend:

$$\Delta Y_{t-1} = \alpha_0 + \emptyset Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i+} e_t$$

The third contains both a constant and a trend:

$$\Delta Y_{t-1} = \alpha_0 + \emptyset Y_{t-1} + \alpha_0 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-1+} e_t$$

In *all three cases* if the value of the underlying entity persists, then for the equation to hold $\emptyset = 1$ All else will lead to net of 0[4].

If we difference using real data rather than artificial data we can see that there were, for example, two occasions when the constant rate of CPI led to zero value in differencing for Monthly Data on CPI in 2015 (March-April and September-October). This is set out in Table 2:

Table 2: Monthly Data on Consumer Price Index %: 2015

Year	CPI% (Y_t)	ΔYt	Year	CPI% (Y_t)	ΔY_t
January	0.5	-	**July**	0.5	0.2
February	0.4	- 0.1	**August**	0.4	-0.1
March	0.3	-0.1	**September**	0.2	-0.2
April	0.3	**0.0**	**October**	0.2	**0.0**
May	0.4	0.1	**November**	0.4	0.2
June	0.3	-0.1	**December**	0.5	0.1

Source: Office for the National Statistics 2020

Inflation was under-shooting the target in 2015. However, being *at* target would not have solved the problem illustrated in Table 1. It is the continual attempt to achieve the target rather than necessarily achieving it that is fundamental to the paradox problem. As noted in the introduction, in reality inflation targets are rarely achieved. For example, in the UK, between December 2003 (when the current 2% target for CPI was established for the Bank of England) and March 2020, only 11 monthly observations coincide with the 2% value. Moreover, this 2% value did not persist into subsequent periods, except in August and September 2007 (ONS, 2020).

3. A practical paradox?

Table 1 and Table 2 both support the inference that the existence of a targeted policy objective (a positive value) is problematic and we can maintain that in the case of persistent achievement of a targeted policy objective the outcome of that policy will work at cross purposes to the transformation of the series typically undertaken for statistical analysis. By way of counter-argument one may respond

[4]To be clear, the problem persists even if alternative specifications or approaches to unit root testing are applied. For example, Phillips and Perron (1988), Ng and Perron (2001), Shahbaz et al (2018), Omay et al (2020).

that quantitative macro models have long been able to handle stationary shocks, and are informed by non-stationary aggregate variables. However, this is disputed (e.g. Stock, 1991). Moreover, one might argue that model solutions suppress or disguise the paradox problem rather than resolve it.[5]

To be clear, an unintended consequence of policy is non-stationarity and policy is *permanently* seeking to *actualise* conditions that will have this consequence. Clearly, the underlying problem is one that persists in so far as the policy objective is retained. On this basis, a person with expertise in the field, might initially think the problem devolves to matters regarding trend-stationary (TS) versus difference-stationary (DS) solutions to a dataset problem (for example, contrast Nelson and Plosser, 1982 and Mitchell, 1993). However, this *does not* directly address the problem we are concerned with because it does not address the problem of a quantified objective as the persistent policy context. It simply creates a further problem because of different ways to address datasets.

In any case, the *creation* of a unit root situation leads to basic problems of interpretation of the stationary and non-stationary data series and of the results of any models constructed in terms of the latter. There is no guarantee that even a 'well-specified' model will have the same signs and values as the original dataset. In fact, this is unlikely. Of course, central bank policy is evidence-based and does not rely on a single source of evidence. However, if a standard approach to econometric tests creates divergent outcomes based on how data is processed, then applied methods that are basic to economics' contribution to the policy decision can be as much a source of confusion as clarity.

It is extremely important to understand what the basis of this confusion is. It is not *merely* a 'spurious model' problem, but rather a problem of data conformation creating different outcomes and indeterminacy. Economics seeks to be a science, the unspoken justification or authority this provides flows in part from the claim that statistical analysis and the tests applied create clarity. Both claim and authority are undermined if methods are technically 'correct' but practically unclear because of issues that are not reconciled, fixes that are required, and limits that are unacknowledged. 'Rigour' becomes

[5] Other issues might arise though these are mainly of peripheral concern. For example, use of regime switching etc.

questionable (Cartwright and Davis, 2016). The unit root paradox is one instance of this. The point is basic and so can easily be overlooked or put aside as statistical analysis becomes more sophisticated.

However, sophistication is not necessarily a solution to the fundamental problem of the paradox (e.g. co-integration introduces new problems as do Bayesian solutions, see Reiss, 2016, Stock 1991, Huang and Yang, 1996). Moreover, further dilemmas arise for the applied economist or social scientist. What new problems is the applied economist prepared to accept in order to achieve stationarity in the data?

There are many potential examples. Differencing typically leads to loss of information regarding common behaviour of two or more *separate* series, which could have provided insight regarding causal inferences (for detailed discussion see Reiss (2015). Busetti (2009) establishes that the magnitude of the initial condition of a highly persistent process affects the properties of stationarity tests applied to the first differences, with 'relatively large' initial conditions, implying non-negligible oversizing. This brings into question the desirability of first difference stationarity. As Lee (1996) notes, achieving a 'correct size' for stationarity tests using optimal lag selection and pre-whitening procedures is at the expense of model explanatory power. As a final point, and parallel to the argument of this brief paper, Gadea and Mayoral (2006) establish that the usual procedure of fitting an AR (k) process to the data and identifying a value of the sum of the AR coefficients close to 1 with the existence of an (integer) unit root can easily lead to persistent overestimation[6]. There are, therefore, multiple issues arising around stationarity and statistical procedure. Rose states a parallel point of context:[7]

> [T]here does not seem to be any statistical evidence inconsistent with the existence of a unit root in GNP. The argument that there is no economic reason to expect a unit root seems weak when confronted with this fact. Looking for the key under the light when there is every

[6] While considering the inflation rates of twenty-one OECD countries which were modelled as fractionally integrated (FI) processes they also showed that FI can appear in inflation rates after aggregating individual prices from firms that face different costs of adjusting their prices. Furthermore, they provide robust empirical evidence supporting the FI hypothesis using both classical and Bayesian techniques.

[7] The second paradox was that one would expect that substituting out exogenous and other endogenous variables would leave one with a univariate equation with much richer dynamics than those actually found. Thirdly, since GNP is the aggregate of components widely accepted to be richly dynamic, one has further reason to believe that the dynamics in the equation are surprisingly simple. Forth, quarterly and annual GNP have remarkably similar time-series processes, even though there is no reason to expect this (See Rose, 1986).

> indication that it was not dropped there, does not seem to be the way to make progress in science. However, the fact that GNP (and many other macroeconomic time-series) likely does have a unit root even though there is no theoretical reason to expect it, surely constitutes an intriguing paradox for macroeconomists.

Consider the policy ramifications. As noted, macroeconomic policymakers and central banks in particular rarely achieve targets and seem profoundly incapable of *consistently* achieving them (a different issue than how useful are these targets). However, the paradox remains a perennial problem *in potentia*. As a 'paradox' it cannot, of course, be stated formally as a construct leading to two mutually incompatible outcomes or consequences (along the lines of Russell's paradox posed to Frege and resolved or 'disallowed' within ZFC), but is worthy of consideration in so far as it reveals a practical problem or limitation created by the very purpose of policy and its relation to or dependence on statistical analysis.[8] Central bank policy is only as good as the evidence it draws on and the principles it expresses. The evidence it draws on is not neutral, it derives from the way data is processed to become evidence.

4. By way of conclusion: philosophical implications

If in concluding we return to the underlying point that an unintended consequence of policy is non-stationarity and policy is permanently seeking to actualise conditions that will have this consequence, then we can draw attention to several additional issues. The mismatch between statistical analysis and policy goal is one rooted in the malleability of economic reality and its complexity. This is a basic issue regarding the nature of economic reality. In terms of statistics, transforming data to achieve stationarity is in effect an attempt to construct data in ways that do not obviously conform to the nature of the world from which the data initially derives (Lawson, 1997; Reiss, 2015). This is a problem that statistical analysis and method has struggled with since the inception of the Cowles Commission. For example, Hendry (1995) makes the point that ultimately method must express something about the 'data

[8] For background issues in the philosophy of mathematics see for example, Shanker ed. 2003; Button & Walsh, 2015; Rieger, 2002. For central banking complexity and instability problems in general see Morgan, 2009. ZFC refers to Zermelo-Fraenkel set theory (ZFC because it includes the 'axiom of choice' and this is standard).

generating process'. Debate regarding these underlying issues is extremely important and, despite the sophistication of much of the literature on statistical methods in-and-of-themselves (e.g. Keane, 2010), is typically underdeveloped in this broader context of purpose and fit (see Freedman 2010; Gillies 2000).

To be clear, in broad context the existence of the paradox of stationarity should not be confused with any of several other superficially 'laws'. According to Strathern (2010, p. 310) Goodhart's law states, 'When a measure becomes a target, it ceases to be a good measure'. That is, it will break down, because external assimilation of the instrument undermines its effectiveness. The Lucas Critique (1976) implicitly expresses something similar in a quite different conceptual context (and without Lucas himself actually recognizing the full ramifications). That is, policy changes systematically alter the micro-foundations that econometric models are intended to express. Campbell's law (1979, p. 85) states that any quantitative indicator used for social decision-making will become subject to corruption pressures liable to distort the processes the indicator is intended to monitor.

All three of these 'laws' (or more accurately related principles or problems) are, for all intents and purposes, instances of adaption where organizations and agents learn and modify their behaviour, in each case responding, managing, capturing or subverting the intention stated as the initial goal. Our point is more basic and not directly concerned with the learning characteristics of those to whom an objective is directed. It presupposes the complexity of economic reality, in particular the complexity of the environment of banking and finance. This is implicit to the 'data generating process'. However, though the paradox has implications for agents and organizations, it exists because of the existence of the quantified objective of policy *irrespective* of the specific characteristics of assimilation by those agents and organizations.[9] There is *prima facie* divergence between the statistical requirement of stationarity and any macroeconomic policy objective that involves a target. When achieved, any such

[9] To be clear, whilst one might choose to take a Bayesian point of view regarding this issue in terms of agent activity through time this does not alter the problem as posed in this paper, it opens up a different set of issues of the role and status of information and change (conceptually) and (analytically) compatibility with testing and distributions. Complex systems of course also open up many other facets of inquiry, including the multiple responses of distinct groupings of agents (since one reason that data of all kinds can be non-stationary is likely to be heterogeneity and learning among agents and thus changes to the way they respond to any structured situation).(Derbyshire, 2016.) We thank an anonymous reviewer for raising these issues.

target will necessarily result in a unit root. As such, an unintended consequence of successful policy is non-stationarity, which means policy is permanently seeking to actualise conditions that are at cross-purposes with typical statistical requirements. We would suggest this requires economists also to be learning lessons.

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