Design of an Optimal Preview Controller for Linear Discrete-Time Periodic Systems

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Abstract: In this paper, the preview tracking control problem for linear discrete-time periodic systems is considered. Firstly, to overcome the difficulty arising from periodicity of the system, the linear discrete-time periodic system is transformed into an ordinary time-invariant system by lifting method. Secondly, the difference between a system state and its steady-state value is used to derive an augmented system instead of the usual difference between system states. Then, the preview controller for the augmented system is proposed by the preview control theory, which solves the preview tracking control problem for the periodic systems. Moreover, an integrator is introduced to ensure that the output can track the reference signal without static error. Finally, the obtained results are illustrated by the simulation examples.

Key word: preview control; periodic system; lifting method; Riccati equation; augmented system

1. Introduction

A linear discrete-time periodic system is a special kind of time-varying systems, which plays a very important role in both theory and practice. Due to the significance of the linear discrete-time periodic systems, in recent years, a growing interest has been attracted on the control of periodic systems. Lv and Zhang (2016) discussed the periodic Luenberger-type observers design problem for linear discrete-time periodic systems. Lovera *et al.* (2002) considered the attitude stabilization and disturbance moment attenuation of small satellites with magnetic actuators and proposed a solution to the problem in terms of optimal periodic control (Lovera et al., 2002). Arcara *et al.* (2000) represented an optimal periodic design method of control systems to reduce vibration of the helicopter main rotor (Arcara *et al.*, 2000). A calculation method is proposed in Varga (2004) to design fault detection filters for periodic systems. So far, with regard to the discrete-time periodic systems, the research activities have been extensively used in hard disk drive servo systems, wind turbines and automobile engines (Allen *et al.*, 2011; Pipeleers *et al.*, 2010). In addition, the discrete-time periodic systems have been widely employed to various fields such as aerial systems, communication systems, image compression systems and voice processing (Lv, 2017). However, seldom attention devoted to the preview control of linear discrete-time periodic systems, for which there still are considerable gaps in literatures.

Preview control is a control technique which improves the tracking performance of a system via utilizing the known future information of the reference signals and disturbance signals (Birla *et al.*, 2015; Zhang, 2000). Since Sheriden (1966) first proposed the concept of preview control, the scientific community began a journey to explore it. Recently, preview control has been discussed deeply for generalized systems, stochastic systems and multi-agent systems (Liao *et al.*, 2012; Gershon *et al.*, 2014; Liao *et al.*, 2016). At the same time, the preview control has attracted significant interest in the applied fields, such as automobile active suspension devices (Gohrle *et al.*, 2014; Li *et al.*, 2014), electromechanical valves (Negm *et al.*, 2006; Mianzo *et al.*, 2007) and automobile driving (Chen *et al.*, 2012). Shimmyo *et al.* (2013) studied the application of preview control in the biped walking robots and used the preview control to generate the walking patterns of the biped walking robots. Li *et al.* (2001) has combined a general optimal servo system with preview feed-forward compensation and applied the optimal preview servo system to the design of terrain tracking controller for cruise missiles (Li *et al.*, 2001). In Zhang *et al.* (1996), by using the area errors as the evaluation function of preview control method, an effective method of high-precision path control was proposed to improve the control effects of robots. Ozdemir *et al.* (2013) revealed the application of preview control in wind turbine and demonstrated the benefits of preview wind measurements for turbine control problems. However, to the best of authors' knowledge, most of the existing results are developed for the linear discrete-time systems, whereas very few results have concentrated on periodic systems. Therefore, it is of great necessity to investigate the preview control for periodic systems.

Motivated by the above statements, the design of an optimal preview controller for periodic systems becomes an important topic. This paper investigates the linear discrete-time periodic systems and presents a new design method of optimal preview controller for such a system. Compared with Liao *et al.* (2019), the system in this paper is more general, in which state matrix, input matrix and output matrix are periodic. Firstly, using the lifting procedure in Liao *et al.* (2019), the original system is converted into a time-invariant system. Then, the difference between a state vector and its steady-state value is used to construct the augmented system, which is a different approach to the traditional one. Thus, the preview tracking problem is reduced to an optimal regulation problem. An integrator is introduced to the closed-loop system, which guarantees that the output of the system tracks the reference signal without static error. Finally, a preview controller is designed for the system to improve the quality of the closed-loop system.

The main contributions of this paper can be listed as follows: (1) The system considered in this paper is more general, which contains the periodic system in Liao *et al.* (2019). To be specific, the coefficient matrices of the system in this study are all periodic, which is an extension of the previous studies. (2) Instead of the usual difference method, a new approach is adopted to derive the augmented error system.

(3) A new design technique of preview controller for linear discrete-time periodic systems has been suggested to improve the tracking performance of the system. (4) Two simulation examples are given to illustrate the effectiveness and advantages of the proposed method. Compared with the previous work, such as in Liao *et al.* (2019), the control performance of the proposed method is better than that of the existing method.

2. Problem Formulation

Consider a linear discrete-time system

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k) \end{cases}, \tag{1}$$

where $x(k) \in R^n, u(k) \in R^r, y(k) \in R^m$ represent the state vector, input vector, and the output vector, respectively. $A(k) \in R^{n\times n}, B(k) \in R^{n\times r}, C(k) \in R^{m\times n}$ represent the coefficient matrices of the system. Supposing all coefficient matrices $A(k), B(k), C(k)$ take p as a period, namely,

$$A(k+p) = A(k), B(k+p) = B(k), C(k+p) = C(k).$$

Because of this characteristic, System (1) is called a periodic system.

Denote $A_s = A(k)$, $B_s = B(k)$, $C_s = C(k)$, where $s \equiv k(\text{mod } p)$ ($s = 0,1,2,\cdots,p-1$). Construct the matrices

$$\bar{A} = \begin{bmatrix} 0 & 0 & \cdots & 0 & A_0 \\ 0 & 0 & \cdots & 0 & A_1A_0 \\ 0 & 0 & \cdots & 0 & A_2A_1A_0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & A_{p-1}A_{p-2}\cdots A_0 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B_0 & 0 & 0 & \cdots & 0 \\ A_1B_0 & B_1 & 0 & \cdots & 0 \\ A_2A_1B_0 & A_2B_1 & B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{p-1}A_{p-2}\cdots A_1B_0 & A_{p-1}A_{p-2}\cdots A_2B_1 & A_{p-1}A_{p-2}\cdots A_3B_2 & \cdots & B_{p-1} \end{bmatrix},$$

$$\bar{C} = diag(\underbrace{C_0, C_1, C_2, \cdots, C_{p-1}}_{p个}),$$

where $\bar{A}$, $\bar{B}$, $\bar{C}$ are constant matrices of dimensions $(pn)\times(pn)$, $(pn)\times(pr)$, $(pm)\times(pn)$, respectively.

The following assumptions have been made for the system:

A1 $(\bar{A},\bar{B})$ is stabilizable and $(\bar{C},\bar{A})$ is detectable.

A2 $\begin{bmatrix} \bar{A}-I & \bar{B} \\ \bar{C} & 0 \end{bmatrix}$ has full row rank.

A3 Reference signal $R(k)$ is previewable in the future M_R period, namely that the future value $R(k)$ is available in pM_R step. In other words, $R(k), R(k+1), R(k+2), \cdots, R(k+pM_R)$ are available. Additionally, the reference signal satisfies

$$R(k+j)=r, j \geq pM_R+1$$

and

$$\lim_{k\to\infty} R(k)=r,$$

where r is a constant vector.

Define the error signal as

$$e(k)=y(k)-R(k).$$

The purpose of this paper is to design a controller with preview compensation for System (1) such that

$$\lim_{k\to\infty} e(k)=\lim_{k\to\infty}[y(k)-R(k)]=0.$$

3. The Lifting of Periodic Systems

Firstly, fully consider the periodicity of system coefficient matrices, transform the problem into that of time-invariant systems through the lifting method. For any k, $k=pi, pi+1,\cdots, pi+(p-1)$, substituting in System (1), the following formula (2) can be obtained

$$\begin{cases} x(pi+1) = A_0 x(pi) + B_0 u(pi) \\ x(pi+2) = A_1 x(pi+1) + B_1 u(pi+1) \\ x(pi+3) = A_2 x(pi+2) + B_2 u(pi+2) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x(pi+p) = A_{p-1} x(pi+p-1) + B_{p-1} u(pi+p-1) \end{cases} . \tag{2}$$

Actually, (2) is the result that the state equation in (1) which is divided into a group according to a period. Denote

$$\bar{x}(i) = \begin{bmatrix} x(p(i-1)+1) \\ x(p(i-1)+2) \\ x(p(i-1)+3) \\ \vdots \\ x(p(i-1)+p) \end{bmatrix}, \bar{u}(i) = \begin{bmatrix} u(pi) \\ u(pi+1) \\ u(pi+2) \\ \vdots \\ u(pi+p-1) \end{bmatrix}, \bar{y}(i) = \begin{bmatrix} y(p(i-1)+1) \\ y(p(i-1)+2) \\ y(p(i-1)+3) \\ \vdots \\ y(p(i-1)+p) \end{bmatrix}.$$

Applying the method in Yang (2018), (2) can be converted into

$$\bar{x}(i+1) = \bar{A}\bar{x}(i) + \bar{B}\bar{u}(i) .$$

Combining the output equation, the following equation can be obtained:

$$\begin{cases} \bar{x}(i+1) = \bar{A}\bar{x}(i) + \bar{B}\bar{u}(i) \\ \bar{y}(i) = \bar{C}\bar{x}(i) \end{cases} . \tag{3}$$

For all time i, $\bar{A}, \bar{B}, \bar{C}$ are constant matrices, so that System (3) is a time-invariant system. Our aim is to design a preview controller for (1) through the lifted System (3).

Similarly, $R(k)$ and $e(k)$ can also be lifted. The lifting method can be basically described that, from the beginning, each p component is divided into a segment and each segment is arranged into a column vector. Therefore, the reference vector and the error vector can be lifted as

$$\bar{R}(i) = \begin{bmatrix} R(p(i-1)+1) \\ R(p(i-1)+2) \\ R(p(i-1)+3) \\ \vdots \\ R(p(i-1)+p) \end{bmatrix}, \quad \bar{e}(i) = \begin{bmatrix} e(p(i-1)+1) \\ e(p(i-1)+2) \\ e(p(i-1)+3) \\ \vdots \\ e(p(i-1)+p) \end{bmatrix} = \bar{y}(i) - \bar{R}(i).$$

So far, the preview control problem is obtained as follows: design a controller $\bar{u}(i)$ for System (3) such that the output $\bar{y}(i)$ of the closed-loop system can track the reference signal $\bar{R}(i)$ asymptotically, or the error $\bar{e}(i)$ is asymptotically stable to a zero vector.

Then, the usual method in preview theory can be utilized to convert the tracking problem into an adjustment problem by constructing the augmented system. Later, the controller is obtained through the optimal control method.

In fact, if the output of (3) tracks the reference signal asymptotically, there always exist vector $\bar{x}(\infty), \bar{u}(\infty)$ such that

$$\begin{cases} \bar{x}(\infty) = \bar{A}\bar{x}(\infty) + \bar{B}\bar{u}(\infty) \\ \bar{R} = \bar{C}\bar{x}(\infty) \end{cases} . \tag{4}$$

Namely,

$$\begin{bmatrix} \bar{A}-I & \bar{B} \\ \bar{C} & 0 \end{bmatrix} \begin{bmatrix} \bar{x}(\infty) \\ \bar{u}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix} \bar{R} , \tag{5}$$

where $\bar{R}$ represents the steady-state value of $\bar{R}(i)$ at infinity, that is,

$$\bar{R} = [r^T \quad r^T \quad \cdots \quad r^T]^T .$$

Thus, under the assumption A2, $\bar{x}(\infty)$ and $\bar{u}(\infty)$ in Equation (5) exist.

Define

$$\tilde{x}(i) = \bar{x}(i) - \bar{x}(\infty), \quad \tilde{u}(i) = \bar{u}(i) - \bar{u}(\infty), \quad \tilde{y}(i) = \bar{y}(i) - \bar{y}(\infty), \quad \tilde{R}(i) = \bar{R}(i) - \bar{R}$$

Subtract both sides of Equation (4) using Equation (3). Then, the following is obtained:

$$\begin{cases} \bar{x}(i+1) - \bar{x}(\infty) = \bar{A}(\bar{x}(i) - \bar{x}(\infty)) + \bar{B}(\bar{u}(i) - \bar{u}(\infty)) \\ \bar{y}(i) - \bar{y}(\infty) = \bar{C}(\bar{x}(i) - \bar{x}(\infty)) \end{cases} .$$

That is

$$\begin{cases} \tilde{x}(i+1) = \bar{A}\tilde{x}(i) + \bar{B}\tilde{u}(i) \\ \tilde{y}(i) = \bar{C}\tilde{x}(i) \end{cases} \tag{6}$$

Equally, based on the definition of error vector, (7) can be obtained:

$$\bar{e}(i) = \bar{y}(i) - \bar{R}(i) = \bar{C}(\bar{x}(i) - \bar{x}(\infty)) - (\bar{R}(i) - \bar{R}) = \tilde{y}(i) - \tilde{R}(i) . \tag{7}$$

In order to design a satisfied controller, the quadratic performance index for (6) is introduced

$$J=\sum_{i=1}^{\infty}\left[\bar{e}^{T}(i)\bar{Q}\bar{e}(i)+\tilde{u}^{T}(i)\bar{H}\tilde{u}(i)\right] \quad , \tag{8}$$

where $\bar{Q}>0,\bar{H}>0$. Therefore, the original tracking problem is reduced to a preview control problem of (6). The quadratic performance index is given in (8). The previewable reference signal is defined by $\bar{R}(i)$, where M_R is the preview length, that is, at the current time i, the current value and the M_R step future value of the reference signal $\bar{R}(i)$ are available.

4. Construction of Augmented System

Substituting the state equation of (6) into (7) gives

$$\bar{e}(i+1)=\bar{C}\bar{A}\tilde{x}(i)+\bar{C}\bar{B}\tilde{u}(i)-\tilde{R}(i+1) \quad . \tag{9}$$

By utilizing the method in Liao et al. (2003), the following augmented system are obtained by combining (6) and (9) together,

$$\begin{cases} X_0(i+1)=\tilde{A}_0 X_0(i)+\tilde{B}_0\tilde{u}(i)+\tilde{G}_{R0}\tilde{R}(i+1) \\ \bar{e}(i)=\tilde{C}_0 X_0(i) \end{cases} , \tag{10}$$

where

$$X_0(i)=\begin{bmatrix}\bar{e}(i)\\ \tilde{x}(i)\end{bmatrix},$$

$$\tilde{A}_0=\begin{bmatrix}0 & \bar{C}\bar{A}\\ 0 & \bar{A}\end{bmatrix}=\left[\begin{array}{ccccc|ccccc} 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & CA_0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & CA_1A_0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & CA_{p-2}\cdots A_0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & CA_{p-1}A_{p-2}\cdots A_0 \\ \hline 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & A_0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & A_1A_0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & A_{p-2}\cdots A_0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & A_{p-1}A_{p-2}\cdots A_0 \end{array}\right]\in R^{(pm+pn)\times(pm+pn)},$$

$$\tilde{B}_0 = \begin{bmatrix} \bar{C}\bar{B} \\ \bar{B} \end{bmatrix} = \left[\begin{array}{ccccc} CB_0 & 0 & 0 & \cdots & 0 \\ CA_1B_0 & CB_1 & 0 & \cdots & 0 \\ CA_2A_1B_0 & CA_2B_1 & CB_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA_{p-1}A_{p-2}\cdots A_1B_0 & CA_{p-1}A_{p-2}\cdots A_2B_1 & CA_{p-1}A_{p-2}\cdots A_3B_2 & \cdots & CB_{p-1} \\ \hline B_0 & 0 & 0 & \cdots & 0 \\ A_1B_0 & B_1 & 0 & \cdots & 0 \\ A_2A_1B_0 & A_2B_1 & B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{p-1}A_{p-2}\cdots A_1B_0 & A_{p-1}A_{p-2}\cdots A_2B_1 & A_{p-1}A_{p-2}\cdots A_3B_2 & \cdots & B_{p-1} \end{array}\right] \in R^{(pm+pn)\times(pr)}$$

$$\tilde{C}_0 = \begin{bmatrix} I_{pm} & 0_{(pm)\times(pn)} \end{bmatrix} = \left[\begin{array}{cccc|cccc} I_m & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & I_m & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_m & 0 & 0 & \cdots & 0 \end{array}\right] \in R^{(pm)\times(pm+pn)},$$

$$\tilde{G}_{R0} = \begin{bmatrix} -I_{pm} \\ 0_{(pn)\times(pm)} \end{bmatrix} = \left[\begin{array}{cccc} -I_m & 0 & \cdots & 0 \\ 0 & -I_m & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -I_m \\ \hline 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{array}\right] \in R^{(pm+pn)\times(pm)}.$$

In this case, the quadratic performance index function given by (8) can be written as

$$J = \sum_{i=1}^{\infty}\left[X_0^T(i)(\tilde{C}_0^T\bar{Q}\tilde{C}_0)X_0(i) + \tilde{u}^T(i)\bar{H}\tilde{u}(i)\right] = \sum_{i=1}^{\infty}[X_0^T(i)Q_0X_0(i) + \tilde{u}^T(i)\bar{H}\tilde{u}(i)], \tag{11}$$

where

$$Q_0 = \begin{bmatrix} \bar{Q} & 0 \\ 0 & 0 \end{bmatrix} \in R^{(pm+pn)\times(pm+pn)}.$$

So far, the problem is reduced to design an optimal preview controller for (10) which minimizes the quadratic performance index function (11).

It can be seen that the difference of $\tilde{u}(i)$ is not included in (10), so that the sum of $\bar{e}(i)$ is not contained in the control input, which makes the closed-loop system without an integrator. This is not beneficial to eliminate static error. Therefore, an integrator is introduced for this purpose

$$\bar{v}(i+1)=\bar{v}(i)+\bar{e}(i) \quad . \tag{12}$$

Let $\bar{v}(\infty)=\lim_{i\to\infty}\bar{v}(i)$, and define $\tilde{v}(i)=\bar{v}(i)-\bar{v}(\infty)$, (12) can be rewritten as

$$\tilde{v}(i+1)=\tilde{v}(i)+\bar{e}(i) \quad . \tag{13}$$

Later, define $X_1(i)=\begin{bmatrix} X_0(i) \\ \tilde{v}(i) \end{bmatrix}$ and combining (10) and (13) gives

$$\begin{cases} X_1(i+1)=\tilde{A}_1 X_1(i)+\tilde{B}_1\tilde{u}(i)+\tilde{G}_{R1}\tilde{R}(i+1) \\ \bar{e}(i)=\tilde{C}_1 X_1(i) \end{cases} , \tag{14}$$

where

$$\tilde{A}_1=\begin{bmatrix} \tilde{A}_0 & 0 \\ \tilde{C}_0 & I \end{bmatrix}, \tilde{B}_1=\begin{bmatrix} \tilde{B}_0 \\ 0 \end{bmatrix}, \tilde{G}_{R1}=\begin{bmatrix} \tilde{G}_{R0} \\ 0 \end{bmatrix}, \tilde{C}_1=\begin{bmatrix} \tilde{C}_0 & 0 \end{bmatrix}.$$

For the system (14), if J in (11) is considered as the quadratic performance index function, the requirement that the corresponding Riccati has a unique positive solution will not be satisfied (Shinzo *et al.*, 1992). Thus, the quadratic performance index function is revised to

$$\begin{aligned} \tilde{J} &= J+\sum_{i=1}^{\infty}\tilde{v}^T(i)Q_v\tilde{v}(i) \\ &=\sum_{i=1}^{\infty}[X_0^T(i)Q_0X_0(i)+\tilde{u}^T(i)\bar{H}\tilde{u}(i)+\tilde{v}^T(i)Q_v\tilde{v}(i)] \\ &=\sum_{i=1}^{\infty}[X_1^T(i)QX_1(i)+\tilde{u}^T(i)\bar{H}\tilde{u}(i)] \quad , \end{aligned} \tag{15}$$

where $Q=\begin{bmatrix} Q_0 & 0 \\ 0 & Q_v \end{bmatrix}, Q_v>0.$

Due to the fact that $\tilde{J}$ reaches the minimum while J does, the controller can be designed with $\tilde{J}$ (Shinzo *et al.*, 1992).

5. Design of Optimal Preview Controller

Lemma 1(Katayama *et al.*, 1987) $(\tilde{A}_1, \tilde{B}_1)$ is stabilizable if and only if $(\bar{A}, \bar{B})$ is stabilizable and $\begin{bmatrix} I-\bar{A} & \bar{B} \\ \bar{C} & 0 \end{bmatrix}$ has full row rank.

Lemma 2(Katayama *et al.*, 1987) $(Q^{1/2}, \tilde{A}_1)$ is detectable if and only if $(\bar{C}, \bar{A})$ is detectable.

According to Katayama *et al.* (1987), the following theorem can be obtained from the preview control theory.

Theorem 1 Supposing that A1, A2, and A3 hold, the preview controller for System (14) that minimizes the quadratic performance index (15) is given by

$$\tilde{u}(i) = F_0 X_1(i) + \sum_{j=1}^{M_R} F_R(j)\tilde{R}(i+j)$$

$$= F_e \bar{e}(i) + F_x \tilde{x}(i) + F_v \tilde{v}(i) + \sum_{j=1}^{M_R} F_R(j)\tilde{R}(i+j), \qquad (16)$$

where

$$F_0 = \begin{bmatrix} F_e & F_x & F_v \end{bmatrix} = -[\bar{H} + \tilde{B}_1^T \bar{P} \tilde{B}_1]^{-1} \tilde{B}_1^T \bar{P} \tilde{A}_1,$$

$$F_R(j) = -[\bar{H} + \tilde{B}_1^T \bar{P} \tilde{B}_1]^{-1} \tilde{B}_1^T (\xi^T)^{j-1} \bar{P} \tilde{G}_{R1} \quad j = 1, \cdots, M_R,$$

$$\xi = \tilde{A}_1 + \tilde{B}_1 F_0 .$$

$\bar{P}$ is the semi-positive definite solution of Riccati equation

$$\bar{P} = Q + \tilde{A}_1^T \bar{P} \tilde{A}_1 - \tilde{A}_1^T \bar{P} \tilde{B}_1 [\bar{H} + \tilde{B}_1^T \bar{P} \tilde{B}_1]^{-1} \tilde{B}_1^T \bar{P} \tilde{A}_1 \quad .$$

Proof: First, according to Katayama *et al.* (1987), the theorem holds if and only if $(\tilde{A}_1, \tilde{B}_1)$ is stabilizable, $(Q^{1/2}, \tilde{A}_1)$ is detectable and the reference signal is previewable. From Lemma 1 and Lemma 2, $(\tilde{A}_1, \tilde{B}_1)$ is stabilizable if and only if $(\bar{A}, \bar{B})$ is stabilizable and the matrix $\begin{bmatrix} I-\bar{A} & \bar{B} \\ \bar{C} & 0 \end{bmatrix}$ has full row rank. $(Q^{1/2}, \tilde{A}_1)$ is detectable if and only if $(\bar{C}, \bar{A})$ is detectable. Therefore, the theorem holds if A1, A2, and A3 are satisfied. This completes the Theorem 1.

Furthermore, based on (16), $\bar{u}(i)$ can be derived

$$\bar{u}(i)=\bar{u}(\infty)+F_e\bar{e}(i)+F_x[\bar{x}(i)-\bar{x}(\infty)]+F_v(\sum_{s=0}^{i-1}\bar{e}(s)+\bar{v}(0)-\bar{v}(\infty))+\sum_{j=1}^{M_R}F_R(j)(\bar{R}(i+j)-\bar{R})$$

Partition F_e, F_x, F_v and $F_R(j)$ in Theorem 1 as

$$F_e=\begin{bmatrix}F_e^{(1)}\\F_e^{(2)}\\\vdots\\F_e^{(p)}\end{bmatrix}=\begin{bmatrix}F_e(1,1)&F_e(1,2)&\cdots&F_e(1,p)\\F_e(2,1)&F_e(2,2)&\cdots&F_e(2,p)\\\vdots&\vdots&&\vdots\\F_e(p,1)&F_e(p,2)&\cdots&F_e(p,p)\end{bmatrix},$$

$$F_x=\begin{bmatrix}F_x^{(1)}\\F_x^{(2)}\\\vdots\\F_x^{(p)}\end{bmatrix}=\begin{bmatrix}F_x(1,1)&F_x(1,2)&\cdots&F_x(1,p)\\F_x(2,1)&F_x(2,2)&\cdots&F_x(2,p)\\\vdots&\vdots&&\vdots\\F_x(p,1)&F_x(p,2)&\cdots&F_x(p,p)\end{bmatrix},$$

$$F_v=\begin{bmatrix}F_v^{(1)}\\F_v^{(2)}\\\vdots\\F_v^{(p)}\end{bmatrix}=\begin{bmatrix}F_v(1,1)&F_v(1,2)&\cdots&F_v(1,p)\\F_v(2,1)&F_v(2,2)&\cdots&F_v(2,p)\\\vdots&\vdots&&\vdots\\F_v(p,1)&F_v(p,2)&\cdots&F_v(p,p)\end{bmatrix},$$

$$F_R(j)=\begin{bmatrix}F_R^{(1)}(j)\\F_R^{(2)}(j)\\\vdots\\F_R^{(p)}(j)\end{bmatrix}=\begin{bmatrix}F_R(1,1,j)&F_R(1,2,j)&\cdots&F_R(1,p,j)\\F_R(2,1,j)&F_R(2,2,j)&\cdots&F_R(2,p,j)\\\vdots&\vdots&&\vdots\\F_R(p,1,j)&F_R(p,2,j)&\cdots&F_R(p,p,j)\end{bmatrix},$$

where $F_e(i,j)\in R^{r\times m}, F_x(i,j)\in R^{r\times n}, F_v(i,j)\in R^{r\times m}, F_R(i,j)\in R^{r\times m}$ ($i,j=1,2,\cdots,p$).

Then, the following equation can be obtained

$$\begin{bmatrix}u(pi)\\u(pi+1)\\\vdots\\u(pi+p-1)\end{bmatrix}=\bar{u}(\infty)+\begin{bmatrix}F_e^{(1)}\\F_e^{(2)}\\\vdots\\F_e^{(p)}\end{bmatrix}\bar{e}(i)+\begin{bmatrix}F_x^{(1)}\\F_x^{(2)}\\\vdots\\F_x^{(p)}\end{bmatrix}[\bar{x}(i)-\bar{x}(\infty)]+\begin{bmatrix}F_v^{(1)}\\F_v^{(2)}\\\vdots\\F_v^{(p)}\end{bmatrix}\left(\sum_{s=0}^{i-1}\bar{e}(s)+\bar{v}(0)-\bar{v}(\infty)\right)+\sum_{j=1}^{M_R}\begin{bmatrix}F_R^{(1)}(j)\\F_R^{(2)}(j)\\\vdots\\F_R^{(p)}(j)\end{bmatrix}[\bar{R}(i+j)-\bar{R}]$$

$$=\bar{u}(\infty)+\begin{bmatrix}F_e(1,1)&F_e(1,2)&\cdots&F_e(1,p)\\F_e(2,1)&F_e(2,2)&\cdots&F_e(2,p)\\\vdots&\vdots&&\vdots\\F_e(p,1)&F_e(p,2)&\cdots&F_e(p,p)\end{bmatrix}\bar{e}(i)+\begin{bmatrix}F_x(1,1)&F_x(1,2)&\cdots&F_x(1,p)\\F_x(2,1)&F_x(2,2)&\cdots&F_x(2,p)\\\vdots&\vdots&&\vdots\\F_x(p,1)&F_x(p,2)&\cdots&F_x(p,p)\end{bmatrix}[\bar{x}(i)-\bar{x}(\infty)]$$

$$+\begin{bmatrix}F_v(1,1)&F_v(1,2)&\cdots&F_v(1,p)\\F_v(2,1)&F_v(2,2)&\cdots&F_v(2,p)\\\vdots&\vdots&&\vdots\\F_v(p,1)&F_v(p,2)&\cdots&F_v(p,p)\end{bmatrix}\sum_{s=0}^{i-1}\bar{e}(s)+\begin{bmatrix}F_v(1,1)&F_v(1,2)&\cdots&F_v(1,p)\\F_v(2,1)&F_v(2,2)&\cdots&F_v(2,p)\\\vdots&\vdots&&\vdots\\F_v(p,1)&F_v(p,2)&\cdots&F_v(p,p)\end{bmatrix}\left(\bar{v}(0)-\bar{v}(\infty)\right)$$

$$+\sum_{j=1}^{M_R}\begin{bmatrix} F_R(1,1,j) & F_R(1,2,j) & \cdots & F_R(1,p,j) \\ F_R(2,1,j) & F_R(2,2,j) & \cdots & F_R(2,p,j) \\ \vdots & \vdots & & \vdots \\ F_R(p,1,j) & F_R(p,2,j) & \cdots & F_R(p,p,j) \end{bmatrix}[\bar{R}(i+j)-\bar{R}],$$

where $\bar{u}(\infty)$, $\bar{x}(\infty)$, and $\bar{v}(\infty)$ represent the steady-state values of $\bar{u}(i), \bar{x}(i)$, and $\bar{v}(i)$ at infinity, respectively.

According to the definitions of $\bar{e}(i)$, $\bar{x}(i)$, and $\bar{R}(i)$, the above formula can be written by component as:

$$u(pi+q-1)=\bar{u}_q(\infty)+\sum_{j=1}^{p}F_e(q,j)e(p(i-1)+j)+\sum_{j=1}^{p}F_x(q,j)\left[x(p(i-1)+j)-\bar{x}_j(\infty)\right]$$
$$+\sum_{j=1}^{p}F_v(q,j)(\sum_{s=0}^{i-1}e(p(s-1)+j)+\bar{v}_j(0)-\bar{v}_j(\infty))+\sum_{j=1}^{M_R}\sum_{s=1}^{p}F_R(q,s,j)[R(p(i+j-1)+s)-\bar{R}(s)]$$
$$q=1,2,\cdots,p.$$

Thus, Theorem 2 is concluded.

Theorem 2 Supposing that A1, A2, and A3 hold, the preview controller for System (1) is:

$$u(pi+q-1)=\bar{u}_q(\infty)+\sum_{j=1}^{p}F_e(q,j)e(p(i-1)+j)+\sum_{j=1}^{p}F_x(q,j)\left[x(p(i-1)+j)-\bar{x}_j(\infty)\right]$$
$$+\sum_{j=1}^{p}F_v(q,j)(\sum_{s=0}^{i-1}e(p(s-1)+j)+\bar{v}_j(0)-\bar{v}_j(\infty))+\sum_{j=1}^{M_R}\sum_{s=1}^{p}F_R(q,s,j)[R(p(i+j-1)+s)-\bar{R}(s)]$$
$$q=1,2,\cdots,p \quad ,(17)$$

where $\bar{u}_j(\infty)$ and $\bar{x}_j(\infty)$ represent the *j*-th component of the steady-state values of the control input and state vector at infinity, respectively. $\bar{R}(s)$ represents for the s-th component of the steady-state value $\bar{R}$.

While A2 holds, the equation (5) has solutions. Thus, $\bar{u}(\infty)$ and $\bar{x}(\infty)$ can take one of its solutions arbitrarily. In application problems, sometimes, for the sake of simplicity, $\bar{u}(\infty)$ and $\bar{x}(\infty)$ can be assigned directly. The numerical simulation shows that it does not affect the output response of the system in this way. Because the identical equation of $\bar{v}(\infty)$ can be obtained by letting $i \to \infty$ in both sides of (13), $\bar{v}(\infty)$ can also be assigned according to the practical condition.

In (17), $\sum_{j=1}^{p} F_e(q,j)e(p(i-1)+j)$ is the tracking error signal compensation; $\sum_{j=1}^{p} F_x(q,j)\left[x(p(i-1)+j)-\bar{x}_\infty(j)\right]$ is the state feedback; $\sum_{j=1}^{p} F_v(q,j)\sum_{s=0}^{i-1} e(p(s-1)+j)$ is an integrator; $\sum_{j=1}^{M_R}\sum_{s=1}^{p} F_R(q,s,j)[R(p(i+j-1)+s)-\bar{R}(s)]$ is the preview feed-forward compensation.

6. Numerical Simulation

In this section, two numerical examples are provided to evaluate our design method. The first one compares the controller performance with different preview lengths in order to illustrate the benefits of the controller with preview compensation. The second one compares the proposed method with the method of Liao *et al.*(2019) based on the same preview length, and the corresponding results are presented.

Example 1

Consider linear discrete-time periodic System (1) with the period of $p=3$. Supposing that the coefficient matrices of System (1) are

$$A_0=\begin{bmatrix}0.5 & 0.02\\ 0.1 & 1.1\end{bmatrix}, A_1=\begin{bmatrix}1.5 & 0.1\\ 1 & 0.02\end{bmatrix}, A_2=\begin{bmatrix}0.01 & 0.6\\ 1.3 & 0.5\end{bmatrix}$$

$$B_0=\begin{bmatrix}6\\ 2\end{bmatrix}, B_1=\begin{bmatrix}-0.999\\ 2\end{bmatrix}, B_2=\begin{bmatrix}1\\ 0.88\end{bmatrix},$$

$$C_0=[1.01 \quad 0], C_1=[1 \quad 0], C_2=[0.999 \quad 0]$$

Select $\bar{Q}=\begin{bmatrix} 4 & 0 & 0.2 \\ 0 & 1 & 0 \\ 0.2 & 0 & 1 \end{bmatrix}, \bar{H}=diag(0.1,0.5,0.5), Q_v=diag(0.1,0.1,0.1)$.

By calculating, it indicates that the assumptions A1-A3 are satisfied. Hence, the corresponding augmented system verifies all the conditions of Theorem 1. After solving the Riccati equation by Matlab, the gain matrices F_e, F_x, F_v can be obtained

$$F_0=\begin{bmatrix} -0.0188 & -0.0063 & -0.0023 & 0 & 0 & 0 & 0 & -0.8287 & -0.0064 & -0.0188 & -0.0063 & -0.0023 \\ -0.1414 & 0.1604 & -0.0388 & 0 & 0 & 0 & 0 & -0.0698 & 0.0497 & -0.1414 & 0.1604 & -0.0388 \\ 0.2213 & -0.1398 & -0.1782 & 0 & 0 & 0 & 0 & 0.0640 & -0.0463 & 0.2213 & -0.1398 & -0.1782 \end{bmatrix}$$

$$F_e=\begin{bmatrix} -0.0188 & -0.0063 & -0.0023 \\ -0.1414 & 0.1604 & -0.0388 \\ 0.2213 & -0.1398 & -0.1782 \end{bmatrix}$$

$$F_x=\begin{bmatrix} 0 & 0 & 0 & 0 & -0.8287 & -0.0064 \\ 0 & 0 & 0 & 0 & -0.0698 & 0.0497 \\ 0 & 0 & 0 & 0 & 0.0640 & -0.0463 \end{bmatrix}$$

$$F_v=\begin{bmatrix} -0.0188 & -0.0063 & -0.0023 \\ -0.1414 & 0.1604 & -0.0388 \\ 0.2213 & -0.1398 & -0.1782 \end{bmatrix}$$

The simulation can be performed in three scenarios with the preview length of $M_R=0, M_R=1, M_R=9$. Let the reference signal be

$$R(k)=\begin{cases} 0, & k<22 \\ \frac{1}{20}(k-22), & 22\le k\le 62 \\ 2, & k>62 \end{cases}.$$

According to

$$\begin{cases} \bar{x}(\infty)=\bar{A}\bar{x}(\infty)+\bar{B}\bar{u}(\infty) \\ \bar{R}=\bar{C}\bar{x}(\infty) \end{cases}.$$

In this case, because of both input and output are scalar, equation (5) has a unique solution.

$$\bar{x}(\infty)=\begin{bmatrix}1.9802\\2.1387\\2.0000\\4.3937\\2.0020\\4.2211\end{bmatrix},\bar{u}(\infty)=\begin{bmatrix}-1.3524\\1.1854\\-0.6542\end{bmatrix}.$$

In view of the above three cases, the initial values of the state vector and control input are chosen as $x(-2)=x(-1)=x(0)=\begin{bmatrix}0\\0\end{bmatrix}$, $u(0)=u(1)=u(2)=0$. The initial condition of $\bar{v}(k)$ is $\bar{v}(0)=\begin{bmatrix}0\\0\\0\end{bmatrix}$. The steady-state value of $\bar{v}(k)$ at infinity are chosen as $\bar{v}(\infty)=\begin{bmatrix}-15\\-5.5\\-8\end{bmatrix}$, $\bar{v}(\infty)=\begin{bmatrix}1\\2\\0.02\end{bmatrix}$, $\bar{v}(\infty)=\begin{bmatrix}1\\1.2\\0.15\end{bmatrix}$.

Figure 1 shows that the output response of the given system in the case of $M_R=0, M_R=1, M_R=9$, where the black line represents the reference signal.

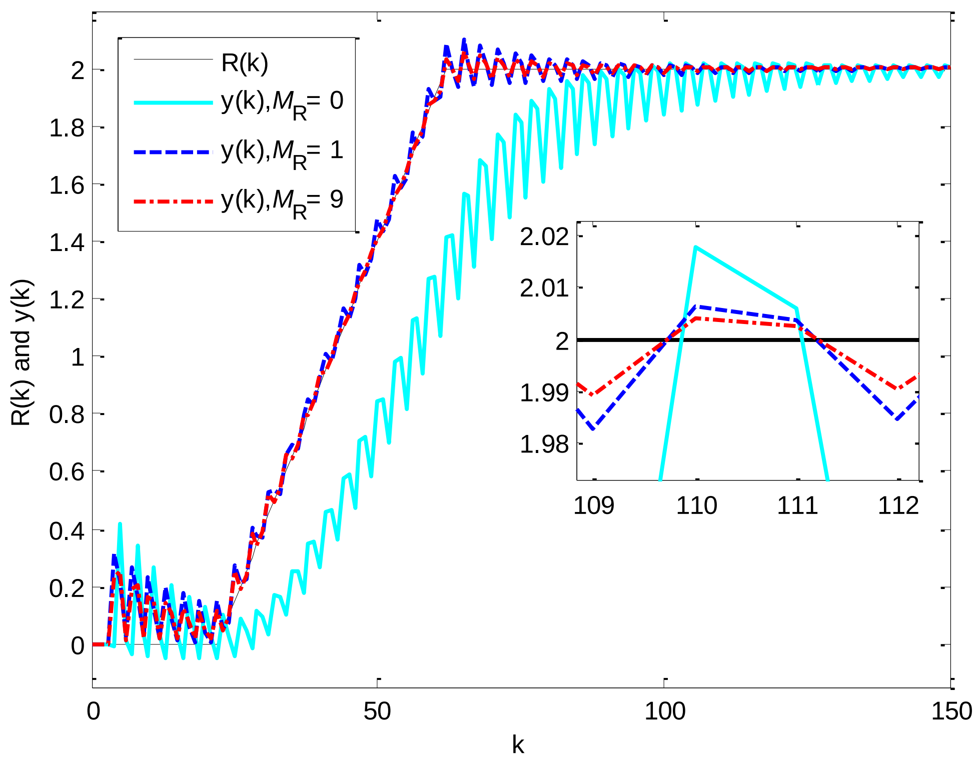

Fig.1 The output response of the closed-loop system

It can be seen from Figure 1 that the controller with preview compensation can make the output track the reference signal more quickly and make the tracking error become smaller than those without preview action.

In order to see the tracking effect more clearly, the tracking error in three cases of $M_R = 0, M_R = 1, M_R = 9$ is shown in Figure 2.

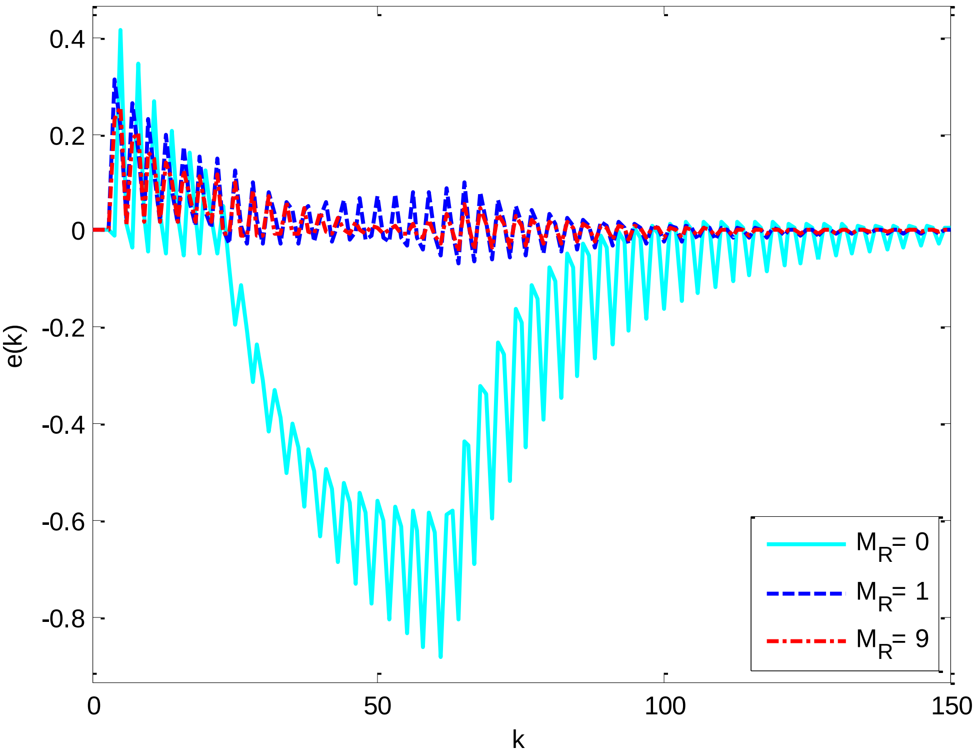

Fig.2 The tracking error

It can be observed from Figure 2 that the controller with preview compensation can reduce the tracking error of the system.

Example 2

Consider linear discrete-time periodic System (1) with the period of $p = 3$. Supposing that the coefficient matrices of System (1) are

$$A_0 = \begin{bmatrix} -2 & 2 \\ 0.2 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 2.9 & 1 \\ -0.1 & 1.3 \end{bmatrix}, A_2 = \begin{bmatrix} 0.89 & 1 \\ 0.01 & 0.99 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 3 \\ 0.05 \end{bmatrix}, B_1 = \begin{bmatrix} 0.01 \\ 1.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 \\ 0.9 \end{bmatrix},$$

$$C_0=[2 \quad 0], C_1=[1 \quad 0], C_2=[3 \quad 0]$$

Select $\bar{Q}=\begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \bar{H}=diag(0.1,0.5,0.5), Q_v=diag(0.1,0.1,0.1)$.

The assumptions A1-A3 are satisfied by calculation. Hence, the corresponding augmented system verifies all the conditions of Theorem 1.

The reference signal is set as

$$R(k)=\begin{cases} 0, & k<22 \\ \frac{1}{20}(k-22), & 22 \le k \le 62 \\ 2, & k>62 \end{cases}$$

In order to illustrate the effectiveness of the proposed method in this paper, it is compared with the method in Liao *et al.* (2019). To facilitate the comparison, the following two cases are taken into consideration. The simulation can be performed in the scenarios with the preview length of $M_R=9$.

Case 1. (Proposed method $M_R=9$)

Solving the Riccati equation gives

$$F_0=\begin{bmatrix} -0.0106 & -0.0200 & -0.0026 & 0 & 0 & 0 & 0 & 0.6385 & -0.7816 & -0.0106 & -0.0200 & -0.0026 \\ 0.0310 & 0.1754 & -0.0699 & 0 & 0 & 0 & 0 & -0.2463 & -1.0331 & 0.0310 & 0.1754 & -0.0699 \\ 0.2156 & -0.2464 & 0.0543 & 0 & 0 & 0 & 0 & -0.0829 & -0.3349 & 0.2156 & -0.2464 & 0.0543 \end{bmatrix}$$

$$F_e=\begin{bmatrix} -0.0106 & -0.0200 & -0.0026 \\ 0.0310 & 0.1754 & -0.0699 \\ 0.2156 & -0.2464 & 0.0543 \end{bmatrix}$$

$$F_x=\begin{bmatrix} 0 & 0 & 0 & 0 & 0.6385 & -0.7816 \\ 0 & 0 & 0 & 0 & -0.2463 & -1.0331 \\ 0 & 0 & 0 & 0 & -0.0829 & -0.3349 \end{bmatrix}$$

$$F_v=\begin{bmatrix} -0.0106 & -0.0200 & -0.0026 \\ 0.0310 & 0.1754 & -0.0699 \\ 0.2156 & -0.2464 & 0.0543 \end{bmatrix}$$

According to

$$\begin{cases} \bar{x}(\infty) = \bar{A}\bar{x}(\infty) + \bar{B}\bar{u}(\infty) \\ \bar{R} = \bar{C}\bar{x}(\infty) \end{cases}.$$

In this case, because of both input and output are scalar, equation (5) has a unique solution.

$$\bar{x}(\infty) = \begin{bmatrix} 1.0000 \\ -0.9015 \\ 2.0000 \\ -1.1052 \\ 0.6667 \\ -1.1108 \end{bmatrix}, \bar{u}(\infty) = \begin{bmatrix} 1.5183 \\ 0.1516 \\ -0.0407 \end{bmatrix}.$$

Case 2. (Liao et al. 2019 $M_R = 9$ **)**

Solving the Riccati equation gives

$$F_0 = \begin{bmatrix} -0.0153 & -0.0697 & -0.0145 & 0 & 0 & 0 & 0 & 0.6373 & -0.7866 \\ 0.0060 & 0.5637 & -0.2123 & 0 & 0 & 0 & 0 & -0.2289 & -0.9610 \\ 0.4857 & -0.6887 & 0.1688 & 0 & 0 & 0 & 0 & -0.1371 & -0.5589 \end{bmatrix}$$

$$F_e = \begin{bmatrix} -0.0153 & -0.0697 & -0.0145 \\ 0.0060 & 0.5637 & -0.2123 \\ 0.4857 & -0.6887 & 0.1688 \end{bmatrix}$$

$$F_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.6373 & -0.7866 \\ 0 & 0 & 0 & 0 & -0.2289 & -0.9610 \\ 0 & 0 & 0 & 0 & -0.1371 & -0.5589 \end{bmatrix}.$$

In view of the above cases, the initial values of the state vector and control input are chosen as $x(-2) = x(-1) = x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $u(0) = u(1) = u(2) = 0$. Let the initial condition of $\bar{v}(k)$ be $\bar{v}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and the steady-state value of $\bar{v}(k)$ at infinity be $\bar{v}(\infty) = \begin{bmatrix} -0.2 \\ 0.1 \\ -0.06 \end{bmatrix}$.

In the following, the simulation results of the proposed controller are illustrated in comparison with the work of Liao *et al.* (2019). Figure 3 shows the output response of the given system in the above two cases with the preview length of $M_R = 9$, where the black line represents the reference signal.

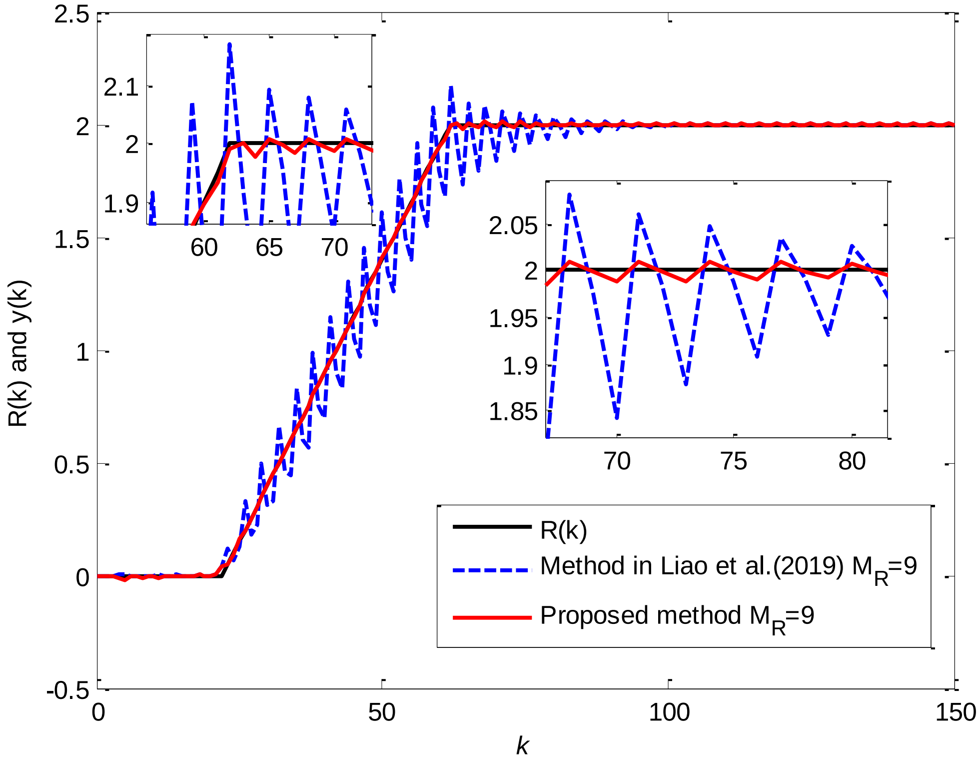

Fig.3 The output response of the closed-loop system

Viewing from the Figure 3, the controller in this paper makes the output track the desired reference signal faster and makes the tracking error become smaller than those in Liao *et al.* (2019). Moreover, under the same preview length, the tracking performance of the proposed method is better than that of Liao *et al.* (2019).

In order to see the tracking effect more clearly, the tracking errors are shown in Figure 4.

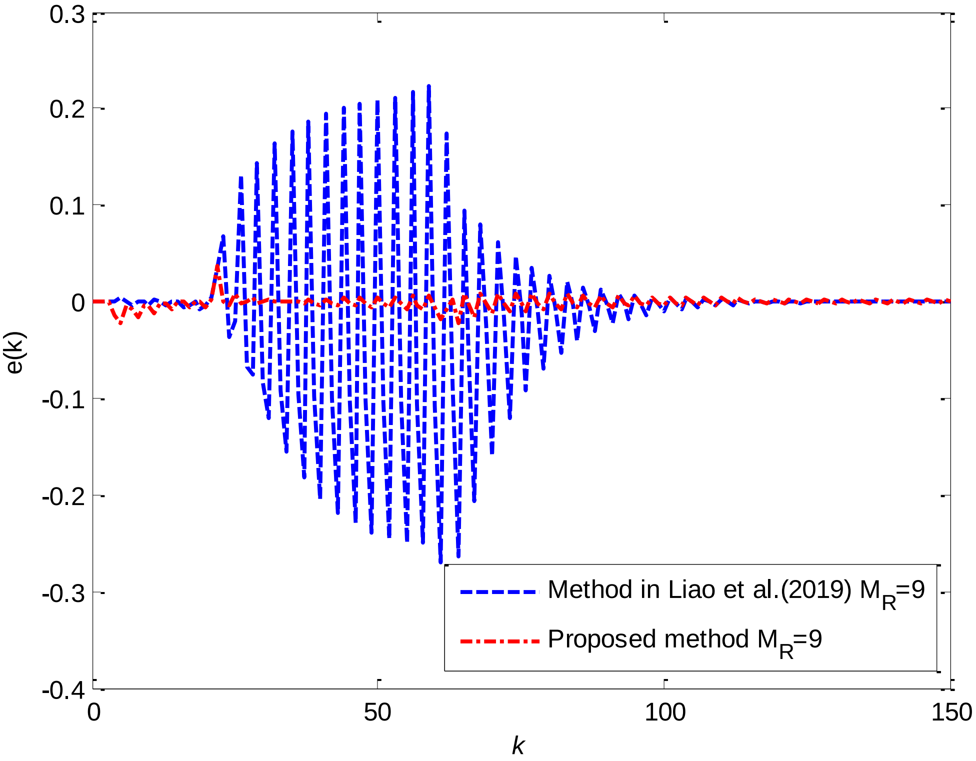

Fig.4 The tracking error

Figure 4 reveals that the tracking error using the suggested method is smaller than that of Liao *et al.* (2019), thereby demonstrating the advantages and effectiveness of the proposed controller approach.

7. Conclusion

In this paper, a new method to design an optimal preview controller for linear discrete-time periodic systems is provided. Combining the preview control method with the lifting method of periodic systems, the augmented system is constructed by the difference between the state and its steady-state value. In order to guarantee that the output of the closed-loop system tracks the reference signal without static error, an integrator is introduced. Compared with the previous work, the simulation results show the effectiveness and merits of the suggested control method.

Declaration of conflicting interest

The authors declare that there is no conflict of interest.

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