Design optimisation of reinforced concrete pile foundation using generalised reduced gradient algorithm

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Abstract

Purpose – The purpose of this study is to find the optimum design of Reinforced Concrete (RC) pile foundation to enable efficient use of structural concrete with greater consequences for global environment and economy.

Design/methodology/approach – A non-linear optimisation technique based on the Generalised Reduced Gradient (GRG) algorithm was implemented to find the minimum cost of RC pile foundation in frictional soil. This was achieved by obtaining the optimum pile satisfying the serviceability and ultimate limit state requirements of BS 8004 and EC 7. The formulated structural optimisation procedure was applied to a case study project to assess the efficiency of the proposed design formulation.

Findings – The results prove that the GRG method in Excel solver is an active, fast, accurate and efficient computer programme to obtain optimum pile design. The application of the optimisation for the case study project shows up to 26% cost reduction compared to the conventional design.

Research limitations/implications – The design and formulation of design constraints will be limited to provisions of BS 8004 and EC 7.

Practical implications – Since the minimum quantity of concrete was attained through optimisation, then minimum cement will be used and thus result in minimum CO_2 emission. Therefore, the optimum design of concrete structures is a vital solution to limit the damage to the Earth's climate and the physical environment resulting from high carbon emissions.

Originality/value – The current study considers the incorporation of different soil ground parameters in the optimisation process rather than assuming any pile capacity value for the optimisation process.

Keywords Optimization, BS 8004, Eurocode 7, Frictional soil, Generalized reduced gradient, Pile foundation

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1. Introduction

In 2013, UK metrological office claimed that the average temperatures around the world have risen by 0.75°C (1.4°F) over the last 100 years. About two-thirds of this increase has occurred after 1975 because of the increasing accumulation of greenhouse gases such as Carbon Dioxide (CO_2) in the atmosphere (Rubenstein, 2012). The continual increase in the quantity of greenhouse gases in the atmosphere has resulted in damaging the Earth's climate and physical environment, these facts were further iterated at COP 26, i.e. 2021 United Nations Climate Change Conference in Glasgow (UN, 2021). This creates the need to reduce energy consumption and carbon footprint on the Earth. Meanwhile, human activities such as cement production involving the burning of fossil fuels is a major source of CO_2 emission accounting for about 5% of global CO_2 emissions (Plaza *et al.*, 2020).

Cement is a major constituent of concrete, and it is the second most-consumed substance on Earth after water, growing by 2.5% annually (Gagg, 2014). According to Rubenstein (2012), concrete consumption is expected to rise from 2.55 billion tons in 2006 to 3.7–4.4 billion tons by 2050. Owing to this unceasingly concrete demand rate and high CO_2 emissions from cement production (i.e. the main concrete constituent), efficient use of concrete will reduce greenhouse gas emissions. These motivations to use concrete efficiently are driving the need for radical improvement in sustainable structures and environmental design. Developments in structural design are attained by adopting more efficient techniques like optimisation of materials consumption for the best satisfactory performance (Salajegheh, 1997). Currently, structural concrete, particularly in the deep foundation is inefficiently used in the built environment. Meanwhile, the cost of foundation such as pile accounts for about 25–40% of the total construction cost. Therefore, it is anticipated that pile design optimisation could achieve significant reductions in material usage and cost; consequently, leading to a reduction in low carbon emission resulting from efficient use of concrete. These in addition to the importance of pile foundation in geotechnical engineering for supporting high significance structures necessitate the need to find the best pile foundation design in terms of performance and economy.

Piles are deep foundations used to support imposed loads from large structures (Rajapakse, 2016). Piles can be classified based on their installation effect on soil (i.e. driven or bored pile), load transmission (i.e. end or point bearing, friction or cohesion and the combination of friction and cohesion pile) and type of construction materials (timber, reinforced concrete, steel and composite pile) (BSI, 2004, 2015). In pile foundation, the imposed loads are transmitted to a low-level underlying stratum. The ability of the supporting ground to withstand the transmitted loads through the pile depends on its design. Pile design is influenced by the scale of the imposed loads, subsoil conditions and the allowable settlement (Das, 2007; Rajapakse, 2016). All these factors mentioned are technical requirements that must be considered in the design and construction of piles. Another significant factor is the construction cost. Therefore, piles designers engaged in pile design optimisation to minimise the effort (cost and time) while maximising the benefit (bearing capacity and durability) (Dauda *et al.*, 2019a).

Optimisation is a mathematical technique established by engineers to design and generate products and structures both economically and efficiently (Salajegheh, 1997). Different optimisation methods such as Generalised Reduced Gradient (GRG), Sequential Quadratic Programming (SQP) and Genetic Algorithms (GA) have been developed to manage varieties of problems (Kao, 1998; Pyrz and Zawidzka, 2001). Accordingly, different tools such as Microsoft Excel Solver and Matrix Laboratory (MATLAB) have been used to apply these methods efficiently (Kao, 1998). The comparative studies of these optimisation methods by Yeniay (2005) revealed that GRG and SQP are two of the pre-eminent deterministic optimisation methods while the GA, built upon the principles of evolution perceived in nature is the finest of the stochastic method.

The application of optimisation by structural engineers began in the early 1900s with Michell (1904). Subsequently, extensive researches have been carried out in the field of structural design optimisation. Nevertheless, few of them have focussed on pile design optimisation due to the challenges in calculating the axial capacity of piles. Meanwhile, the review of existing works (Fellenius, 1998; Leung *et al.*, 2010; Liu *et al.*, 2012; Letsios *et al.*, 2014) shows that these studies lacked the details about the soil properties of the sites under consideration in their design and study. Although these studies report the capacity of different sizes of piles, these studies did not recognise the importance of interpreting the ground parameters in pile design. Those pile capacity values were later used in the optimisation formulation and design.

Meanwhile, pile designers only have soil parameters and imposed load at the beginning of the design. The practitioners require guidelines on selecting the type and geometry (length, diameter) of the pile to resist a specific load. Thus, there is a need for some design charts where some preliminary estimates can be made using the available soil data by considering the charts with assumptions that are relatively the same as their design scenario. The current study thus considers the incorporation of different soil ground parameters in the optimisation process to address these issues exhaustively rather than assuming any pile capacity in the formulation of the optimisation process.

Therefore, this study aims to find the optimum design of concrete pile foundation in relation to minimum cost using BS 8004:2015 and EC7 specifications. The objective is to develop some optimum design charts that can be used in designing concrete pile foundations which will enable efficient use of structural concrete with greater consequences for the global environment and economy. GRG method was used in this study because of its high efficiency for non-linear relation like a pile design problem. Excel solver embedded in Microsoft office add-ins was used as the optimisation tool because of its high availability (Dauda *et al.*, 2019a, b). In addition to the introductory section presented here, the overall concept of pile design according to BS 8004 and EC 7 was presented in Section 2. In Section 3, the formulation of pile design optimisation and its application to a case study project was presented. Thereafter, the results of the optimisation process were discussed in Section 4 while the conclusion and recommendation from this study were conferred in Section 5.

2. Pile design according to BS 8004 and Eurocode (EC) 7

Nowadays, larger percentages of load analysis and design of pile foundations are carried out using many commercial software packages. However, not everyone has access to these commercial programmes. Thus, the principle on which pile design is deemed fundamental to all geotechnical engineers. This section presents an overview of the pile design method that simplifies the calculation procedure according to BS 8004 and EC7.

Pile designs are based on the equation that relates soil and rock parameters with shaft friction and end-bearing as shown in Equation (1). Convectional pile design obeys the limit state theory by ensuring that the applied load is less than the ultimate capacity of the pile so that the safe working load of the pile is not exceeded (Equation 2). However, the accuracy of the result from Equation (1) depends on the reliability of the chosen soil strength data (obtained from either field or laboratory tests) and site-specific pile installation methods (Meyerhof, 1976; Terzaghi *et al.*, 2007).

$$R_t = R_s + R_b \tag{1}$$

$$Q < R_t \tag{2}$$

where R_t is the total bearing resistance of the pile (N), R_s and R_b are the resistance of pile shaft (N) and pile base (N), respectively, while Q is the applied load (N).

After the recent review of BS 8004 in 2015, all its provisions were fully compatible with the current version of EC 7. The provisions of both codes allow for factors of safety for global, shaft and base resistance. These factors of safety account for the consequences of differences between the real and calculated value which might occur owing to differences between the actual and assumed ground conditions. In applying the factors of safety, the applied load could be factored directly to obtain the ultimate design load (Equation 3), or the factor of safety could be used in the calculation of the pile bearing resistance (Equation (4) or (5)) (BSI, 2015). Although the two methods are appropriate, applying the factor of safety on both sides will lead to unnecessary conservation and uneconomical design (Bond, 2009)

$$\left(Q = \psi_g G_k + \psi_q Q_k\right) < R_t \tag{3}$$

$$(Q = G_k + Q_k) < \frac{R_t}{f_g} \tag{4}$$

$$(Q = G_k + Q_k) < \left(R_t = \frac{R_s}{f_s} + \frac{R_b}{f_b}\right) \tag{5}$$

where G_k and Q_k are the characteristic permanent action (N) and characteristic variable action (N), respectively in EC7 which corresponds to dead load in BS 8004; ψ_g and ψ_q is the partial factor of safety for permanent action and variable action respectively; f_g, f_s and f_b are the factor of safety for global resistance, shaft resistance and base resistance, respectively.

In this study, the factor of safety will not be applied to the pile bearing resistance because the design loads from the case study project discussed in Section 3.3 have already been factored to estimate the ultimate design loads for which a pile foundation is required. Therefore, the application of any results from this study will be based on selecting piles foundation for an ultimate design load because the pile bearing resistances have not been factored in.

2.1 Design formulation

The behaviour of pile foundation under loading is often influenced by pile-soil interaction resulting from the soil composition of the site (Maki *et al.*, 2006; Smith and Abebe, 2015). Meanwhile, a typical soil system comprises a multi-layer system of different soil types in reality. However, where characteristics of these soil layers are similar to one another, the pile may be assumed to be embedded in a homogeneous soil with factored characteristics estimated from such individual soil data (Mandal *et al.*, 2012). Therefore, this study considered pile design in a uniform single layer system with and without groundwater as represented in Figures 1a and b. This study assumption is because the case study site to be adopted in this study has an idealised homogenous soil layer with varying water table levels.

Before the calculation of the total pile resistance in any of the two cases illustrated in Figures 1a and b, some dependant design parameters need to be estimated using the values of soil frictional angle as shown in Equations (6)–(10).

$$\xi = 0.75\phi \tag{6}$$

$$\kappa_o = 1 - \sin\phi \tag{7}$$

$$\kappa_a = \tan^2\left(45 - \frac{\phi}{2}\right) \tag{8}$$

$$\kappa_p = \tan^2\left(45 + \frac{\phi}{2}\right) \tag{9}$$

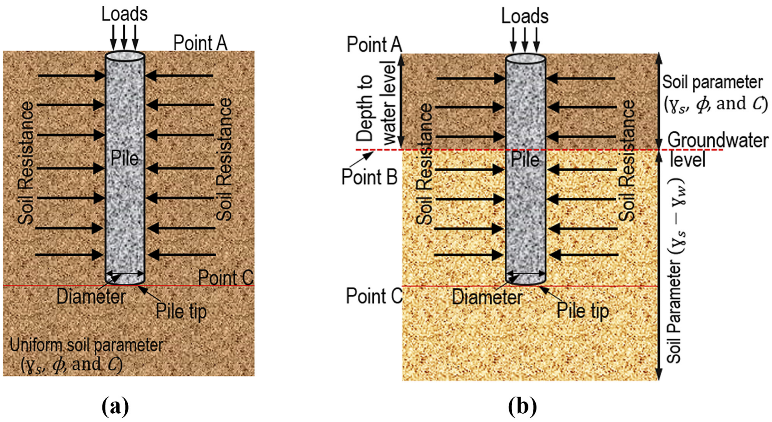

Figure 1. Schematic representation of pile in uniform soil (a) without groundwater (b) with groundwater

$$\kappa = \frac{\kappa_o + \kappa_a + \kappa_p}{3} \tag{10}$$

where ξ and ϕ are the pile skin friction angle and soil friction angle in degree, respectively; κ_o, κ_a, κ_p and κ are the coefficients of Earth pressure at rest, active Earth pressure, passive Earth pressure and Earth pressure against pile shaft, respectively.

Considering Figure 1, the total bearing resistance (R_t) of an individual pile in a single soil layer system without groundwater was calculated using an extension of Equation (1). The resistance of pile shaft (R_s) was calculated using Equations (11)–(13) as follows;

$$R_s = A_s q_s \tag{11}$$

$$A_s = \pi d l \tag{12}$$

$$q_s = \kappa \gamma_s h_c \tan \xi \tag{13}$$

where A_s is the total circumferential area of pile shaft (mm^2), q_s is the ultimate unit shaft resistance/unit skin friction calculated from the ground parameter (N/mm^2), d and l are the pile diameter and length (mm), respectively; γ_s is the unit weight of soil (N/mm^3) and h_c is critical depth (mm) taken as the midpoint of pile length.

Similarly, Equations (14)–(21) were used to estimate the resistance of the pile base (R_b). In this study, the structural analysis isolates the superstructure from the substructure. Hence, the design load considered in this study omitted the self-weight of the pile in the calculation of the action load, as allowed by BSI (2004, 2015). Therefore, the overburden pressure was subtracted from the calculation of R_b in Equation (14) so that the equation becomes as shown in Equation (15).

$$R_b = A_b q_b \tag{14}$$

$$R_b = A_b (q_b - \sigma_v) \tag{15}$$

$$A_b = \frac{\pi d^2}{4} \tag{16}$$

$$q_b = q N_q + C N_c \tag{17}$$

$$q = \gamma_s l \tag{18}$$

$$\sigma_v = q, \text{ where no groundwater presents} \tag{19}$$

$$N_q = e^{\pi \tan\phi} \tan^2\left(45 + \frac{\phi}{2}\right) \tag{20}$$

$$N_c = (N_q - 1)\cot\phi \tag{21}$$

where A_b is the total cross-sectional area of the pile base, C is soil cohesion (N/mm^2), N_q and N_c are bearing capacity factor calculated from the ground parameters, respectively; q is the vertical stress at the pile tip (N/mm^2), q_b is ultimate unit base resistance calculated from the ground parameter (N/mm^2), and σ_v is total overburden pressure at the level of pile base (N/mm^2).

In Figure 1b, a typical case of pile embedded in uniform soil with groundwater, the pile bearing resistance was calculated in two parts. Pile within groundwater (point B to C) and above groundwater (point A to B). The effective vertical stress was calculated by subtracting the effect of pore water pressure from the total vertical stress as follows:

$$q_t = \gamma_s l \tag{22}$$

$$q_w = \gamma_w (l - d_w) \tag{23}$$

$$q' = q_t - q_w \tag{24}$$

where q_t is the total vertical stress at the pile tip (N/mm^2), q_w is the pore water pressure/stress (N/mm^2), γ_w is the unit weight of water (N/mm^3), d_w is the depth to water level (mm) and q' is the effective vertical stress at the pile tip (N/mm^2).

Similarly, the unit skin friction was calculated for both above and below the water table as shown:

$$q_{ss} = \kappa \gamma_s \frac{d_w}{2} \tan\xi \tag{25}$$

$$q_{sw} = \kappa\left(\gamma_s d_w + (\gamma_s - \gamma_w)\frac{l - d_w}{2}\right)\tan\xi \tag{26}$$

$$q_s = q_{ss} + q_{sw} \tag{27}$$

where q_s is the ultimate skin friction resistance, q_{ss} and q_{sw} are the total skin friction resistances above water level and below water level (N/mm^2), respectively.

2.2 Settlement of single pile

For every loaded pile, there is a tendency for the pile to settle into the ground and/or the pile materials to compress due to the load. In each case, this is termed pile settlement. According to BSI (2015), pile settlements are greatly influenced by the ratio of pile length to its diameter. In addition, the Young modulus of the soil is commonly used in the estimation of pile settlement from static loads. With inference from both BS 8004 and EC7, the settlement of a pile foundation may be calculated using any of the following models as appropriate: (1) Numerical models, including the s*emi-empirical approach*; (2) Theory of elasticity; (3) T-Z curves; (4) Hyperbolic stress-strain model; (5) The interaction-factor method (an analysis of the vertical deformation of pile groups); (6) The boundary element method (non-linear analysis of pile groups); (7) Finite element method; and (8) Wave equation analysis.

All these methods are appropriate and conform to both the BS 8004 and EC7. However, comprehensive guidance on the calculation of settlement for single piles using the semi-empirical approach was found in the manual of geotechnical engineering of the Institution of Civil Engineers (ICE, 2012). This makes it to be one of the widely accepted means of estimating pile settlement. As such, this study adopted the semi-empirical approach as given in Equations (28)–(33) because of its simplicity and acceptability (BSI, 2004, 2015).

$$S_{total} = S_a + S_p + S_s \tag{28}$$

$$S_a = \frac{l(R_b + 0.67R_s)}{A_b E_p} \tag{29}$$

$$S_p = \frac{R_b C_p}{dQ_{ult}} \tag{30}$$

$$S_s = \frac{R_s C_s}{lQ_{ult}} \tag{31}$$

$$E_p = 20000 + 0.2f_{ck} \tag{32}$$

$$C_s = C_p\left(0.93 + \frac{0.16l}{d}\right) \tag{33}$$

where S_{total} is the total settlement of a single pile (mm), S_a is the settlement due to axial deformation (mm), S_p is the settlement at pile point due to end-bearing load (mm), S_s is the settlement due to skin friction (mm), f_{ck} is the characteristic strength of concrete (N/mm^2), E_p is the modulus of elasticity of concrete pile (N/mm^2), C_p is the empirical coefficient, C_s is empirical coefficient as a function of C_p and Q_{ult} is ultimate design load (N).

3. Optimisation formulation

In this study, the optimisation of pile foundation design statements was formulated as follows:

(1) Find the set of (n) design variable

$$\{x\} = \{x_1, x_2, \ldots\ldots x_n\} = \{l_{pile}, d_{pile}, A_{steel}\} \tag{34}$$

(2) Minimising the objective function

$$Z = f(\{x\}) = \{x_1, x_2, \ldots\ \ldots x_n\} = \sum\{Cost_{pile}, Cost_{steel}, Cost_{labour}\} \tag{35}$$

(3) Subject to:

- Certain behavioural (implicit) constraints

$$g_i(\{x\}) = g_i\{x_1, x_2, \ldots\ \ldots x_n\} \le 0 \tag{36}$$

- Side (explicit) constraints

$$\{Lower\ x\} \le \{x\} \le \{Upper\ x\} \tag{37}$$

3.1 Design parameters

The independent design parameters are soil cohesion, soil frictional angle, unit weight of soil, unit weight of water, depth to water level, characteristic strength of concrete and steel and applied factored axial load. The variable design parameters which depend on soil friction angle are coefficient of Earth pressure against pile shaft, pile skin friction angle and bearing capacity factor.

3.1.1 Design variables. The design variables were based on the geometry of the pile and the area of steel required viz;

$X1$: Pile diameter (mm)

$X2$: Pile length (mm) and

$X3$: Area of steel required (mm^2)

3.1.2 Objective function. The objective function considered in this study is the minimum cost of the pile (Z). The function for the minimum cost is given as the total sum of the cost of concrete (Z_{conc}), cost of steel (Z_{st}) and labour cost (Z_{lab}). Since the material prices vary from region to region, indicative costing values of (C_{conc} (f_{ck} = 30) = £80/m^3, C_{st} = £800/tonnes and C_{lab} = £230 / m^2/m) were assumed to conduct the total cost formulation.

$$Z_{total} = Z_{conc} + Z_{st} + Z_{lab} \tag{38}$$

$$Z_{conc} = C_{conc} Vol_{conc} \tag{39}$$

$$Z_{steel} = C_{st} Wt_{st} \tag{40}$$

$$Z_{lab} = C_{lab} A_{pile} l_{pile} \tag{41}$$

where Z_{total}, Z_{conc}, Z_{st} and Z_{lab} are the total costs of the pile, concrete, steel and labour, respectively; C_{conc}, C_{st} and C_{lab} are the indicative costing values of concrete, steel and labour, respectively; Vol_{conc} is the volume of concrete, Wt_{st} is the unit weight of steel, A_{pile} and l_{pile} are the area and length of the pile, respectively.

3.2 Design constraints

Design constraints are the limitations of the design, i.e. the condition which must be satisfied in the design (Mohammad and Seyan, 2016; Panagiotis *et al.*, 2021). In this study, the design constraints that must be satisfied to make the results of the pile design optimisation acceptable are;

Pile diameter lower limit	g_1	$X1 \geq 300$
Pile diameter upper limit	g_2	$X1 \leq 1000$
Pile length lower limit	g_3	$X2 \geq 5000$
Pile length upper limit	g_4	$X2 \leq 20000$
Area of steel lower limit	g_5	$X3 \geq A_{st}\min = A_{pile}(0.4\%)$
Area of steel upper limit	g_6	$X3 \leq A_{st}\max = A_{pile}(4\%)$
Minimum ratio of pile length to diameter	g_7	$\frac{X2}{X1} \geq 10$
Maximum ratio of pile length to diameter	g_8	$\frac{X2}{X1} \geq 25$
Maximum skin friction limit	g_9	q_s: $f(X2) \leq 0.11 N/mm^2$
Maximum base resistance limit	g_{10}	q_b: $f(X2) \leq 28.7 N/mm^2$
Minimum allowable total bearing resistance	g_{11}	R_t: $f(X1, X2) \geq 736 kN$
Maximum allowable total bearing resistance	g_{12}	R_t: $f(X1, X2) \leq 4415 kN$
Applied load ≤ pile ultimate bearing resistance	g_{13}	$P_{design} \leq R_t$: $f(X1, X2)$
Applied load ≥ 90% pile ultimate bearing resistance	g_{14}	$P_{design} \geq 0.9R_t$: $f(X1, X2)$
Pile Ultimate Capacity ≥ total pile bearing resistance	g_{15}	$Q_{ult} \geq R_t$: $f(X1, X2)$
Total settlement ≤ allowable settlement	g_{16}	$S_{tot} \leq S_{all} = 10\% d_{pile}$: $f(X1)$
Pile Spacing ≥ thrice pile diameter	g_{17}	*Pile Spacing* $\geq 3d_{pile}$: $f(X1)$

Note(s): $f(X1)$ means the constraint is a function of design variable X1

3.2.1 Explanation of design constraints imposed in this study.

(1) Constraints g_1, g_2, g_3, g_4, g_5 and g_6 are feasible limits (upper and lower) for design variables.

(2) Constraints g_7 and g_8 were based on critical depth which varies between $10d$ and $25d$. A maximum $\frac{l}{d}$ ratio of 25 was used because the maximum value of skin friction resistance and end-bearing capacity is achieved between $20d - 25d$ within the bearing zone (Das, 2007).

(3) Constraints g_9, g_{10}, g_{11} and g_{12} were formulated from allowable capacity limits of the pile.

(4) Constraint g_{13} implies that imposed load must be less than pile capacity to satisfy limit state.

(5) Constraint g_{14} was formulated using engineering judgement because a reasonable adequacy ratio limit must be ensured so that the pile capacity is not underused. Therefore, the study assumed that not less than 90% of the pile resistance should be used.

(6) Constraint g_{15} means that the ultimate capacity of the pile will be greater than pile resistance because the steel provided in the reinforced pile would have augmented the pile strength.

(7) Constraint g_{16} was based on BS 8004 and (EC7) recommendations that the failure criterion for settlement of the pile top is equal to 10% of the pile base diameter.

(8) Constraint g_{17} is from clause 6.1.7 of BS 8004:2015 which says the group effects may be ignored when there are fewer than five piles in the group and the piles are spaced no closer than three times their diameter centre-to-centre (Bond, 2009).

3.3 Optimisation process

GRG embedded in Excel solver was implemented in this study. GRG is very popular in solving non-linear optimisation problems requiring only that the objective function is differentiable such as the cost and volume optimisation presented in this study. The main idea of this method is to solve the non-linear problem dealing with active inequalities. The process involved three key stages as identified in Figure 2. First, the design variables and design parameters separated into a set of independent and dependent parameters are input into excel. Then, all the design equations and constraints are formulated using the equations stated from Equations (1)–(42). Thereafter, a starting point for the design variables is selected to perform the optimisation. Then, the reduced gradient is computed in order to find the minimum in the search direction. This process is repeated until convergence is obtained. A key observation in the process is that different starting point within the limit of the variable gives exactly the same final solution. However, where the initial solution is outside the limit of the variables, the solution did not converge.

3.4 Case study

The structural optimisation procedure developed in this study was implemented on an eleven-storey residential safe tower building to assess the efficiency of the proposed optimisation design formulation. The building is made of Reinforced Concrete (RC) frame construction with overall footprint dimensions of 32.925 m long by 14.325 m wide. The height from ground floor datum to roof ridge is about 40.3 m. The building gravity loads are carried

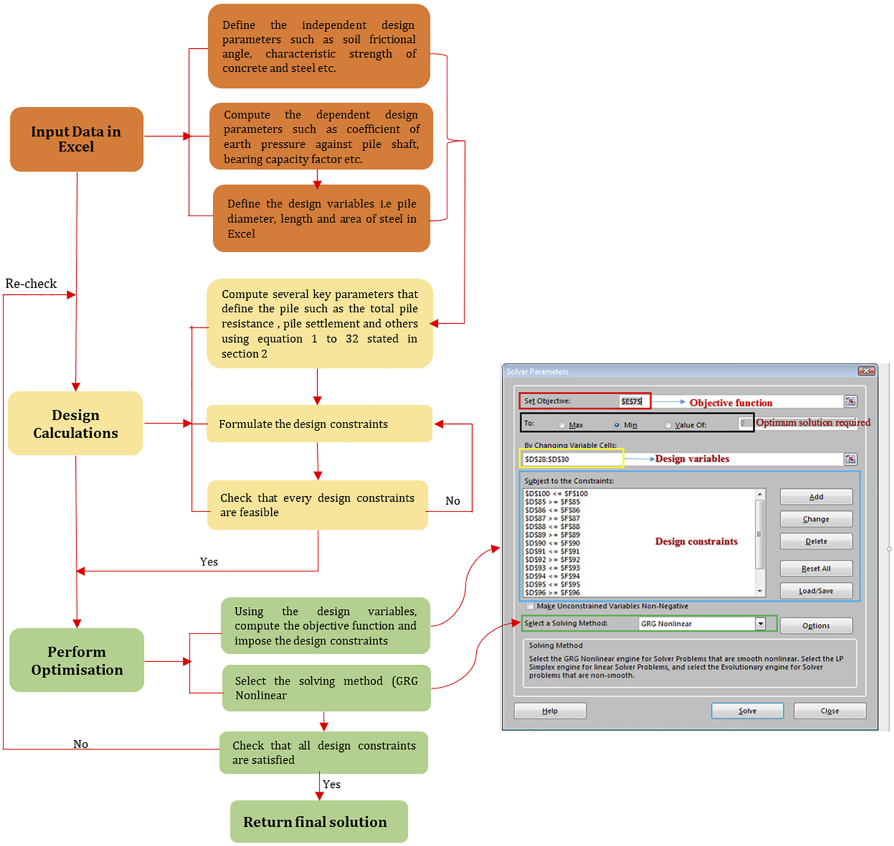

Figure 2.
Optimisation process flowchart

to the foundation through a system of an interconnected grid of slabs, beams and columns. The structure is supported on a pile foundation with the ground floor slab sitting on grade.

3.4.1 Structural analysis of safe tower building. The structural analysis established a separation line between the substructure and the superstructure. The analysis was based on a three-dimensional analytical model of the entire structural framework using yield line analysis or finite element analysis where necessary. The model was analysed for the effects (thrusts, moments, shears, deflections, rotations, reactions, etc.), gravity (dead and live) and lateral (wind and notional) loads. The load cases considered include the dead, imposed, wind loads and the combinations of the three loads cases.

3.4.2 Structural design of safe tower building foundation. Since the analysis was performed separately for the superstructure, the vertical loads on columns and walls required to design the substructure (foundation) were calculated using a load chase down approach. Therefore, the ultimate design loads expected on the pile foundation were as presented in Figure A1. These loads were used to design the pile foundation for the building on frictional soil with an angle of internal friction of 30°. The result from the individual pile optimisation (stage 1 of this study) was used to estimate the pile foundation required for each load group.

The study was carried out using the conventional method to estimate the number and sizes of pile required to carry each load group using Equation (42). The pile group loads were also used in the optimisation process directly by including the g17 constraint. The comparison of the cost of the optimum designs and the conventional design for this study was conducted based on the same cost calculation procedure.

$$\text{Pile required} = \frac{\text{Group ultimate load}}{\text{pile}} \text{ safe working load} \quad (42)$$

4. Results and discussions

First, the condition of optimum design variables achieved from cost optimisation and volume optimisation was assessed. The minimum cost obtained from both volume and cost optimisation for different pile design loads is similar (see Figure A2). This is mainly because the area of steel obtained from both cost and volume optimisation were satisfactory and binding to constraints g5 which is the lower limits of the steel required (minimum reinforcement provided). This makes the result of the optimisation comparable to the conventional design where minimum/nominal reinforcement was also provided. This is a valid design process for pile resisting only axial loads such as the one considered in this design as pointed out in BS 8004:2015. So, this confirms that the cost optimisation from this study resulted from less quantity of concrete which also directly resulted in lesser quantity of steels and subsequently ensuring minimum cost. This clarity is needed to establish the minimum cost is resulting from the minimum use of concrete to meet the aim of this study. The reduction in the steel usage from the optimised design compared to the conventional design is only due to the reduction in the pile size as minimum reinforcement (i.e. $A_{st}\min = (0.4\%)A_{pile}$) was provided in both cases. In addition, constraint g14 which imposes a reasonable adequacy ratio limit of 90% so that the pile capacity is not underused also means that larger concrete pile was not provided unnecessarily resulting in efficient use of concrete which will reduce greenhouse gas emissions.

4.1 Optimum solution

For every result taken from the solver solution, all constraints and optimality conditions specified were satisfied. The optimum pile diameter (Figure 3a) and length (Figure 3b) were obtained for specific pile resistance (R_t) for different soil friction angle (ϕ) decrease with an increase in soil friction angle. This reveals a positive relationship between soil friction angle and the ultimate pile resistance. This indicates that the strength of sandy soils comes mainly from the friction between particles. Thus, the pile capacity in sandy soil depends largely on soil friction angle. This makes soil friction angle the most important design parameter for piles in cohesionless (sandy) soils. As such, this is similar to the conclusions made by Zamri *et al.* (2009) and Johny and Prabha (2014).

Further analysis of the result shows that for $\phi = 10°$, the maximum R_t is 1,881 KN, which was obtained with $d = 1{,}000$ mm and $l = 20$ m that are the upper bound of variables assumed in this study. This explains why the curves for $\phi = 10°$ in Figures 3a–e were truncated barely before 2,000 KN. This means that for a pile to resist more than 2,000 KN axial load in the sandy soil of $\phi \leq 10°$ and $\gamma_s = 18$ N/mm^3, either the pile diameter or length will have to exceed the upper limits assumed in this study (1,000 mm and 20 m respectively).

4.2 Effect of soil frictional angle on the cost of pile

The charts in Figures 3c and d reveal that high frictional soil compares to low frictional soil requires a lesser quantity of materials to resist the same applied load. So, an optimum design of concrete pile foundation in high frictional soil will lead to low cement consumption which ultimately reduced carbon emission. Figure 3e presents the effects of soil friction angle on the

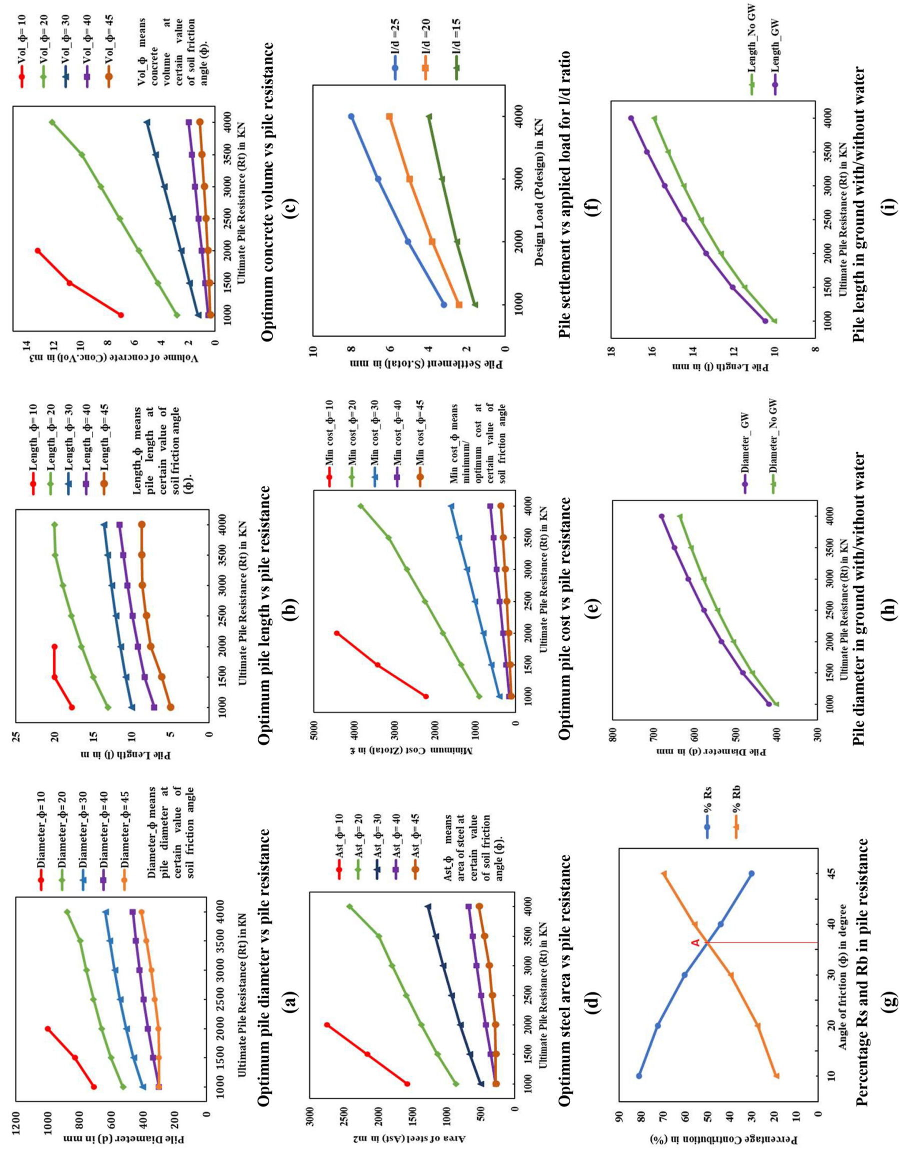

Figure 3.
Optimum results obtained

total cost of pile foundation. The optimum cost reduces with an increase in ϕ-value under a specific applied load. The cost reduction was traced to the decrease in the quantity of materials in Figures 3c and d. It can be deduced from this analysis that sandy soil with a high frictional angle requires a smaller pile to support the same applied load compared to low frictional soil. It is important to highlight that the labour cost of piling work is priced in unit per area per linear metre. This results in a cubic measurement (unit/m^3) which is directly related to the volume of concrete. Hence, a decrease in concrete volume will reduce the total cost. However, engineering judgement is sometimes used to go for a shorter pile but large diameter where the cost of piling per linear metre is high and vice versa to reduce the cost.

4.3 Effect of soil frictional angle on pile settlement

Pile settlement (mm) under each load was also examined for different ϕ-values. The results showed that piles settlement obtained in the optimisation process in this study were far below the acceptable limit of 10% of pile diameter. The maximum settlement obtained in the study is 12.21 << 34.7 mm (10% of 347 mm pile diameter) under 3,000 KN load when $\phi = 45°$. This means that constraint g_{16} is satisfactory but not binding throughout the optimisation process in this study. Figure A3 shows that pile settlements increase with increasing values of ϕ except for only design loads 1,000 and 1,500 kN, where there is a slight decrease in the pile settlement when $\phi = 45°$. Generally, the Young Modulus of soil greatly influences pile settlement under specific design load and the soil Modulus increases with shear strength that depends on ϕ-value as deduced from Mohr-Coulomb theory. This is in agreement with the findings of Gopikrishnan and Varkey (2020).

In addition, Figure 3f reveals that for the same load and angle of internal friction, pile settlement increases with slenderness ratio $\left(\frac{l}{d}\right)$. This shows that pile settlement is in a positive relationship with the $\frac{l}{d}$ ratio.

4.4 Relationship between skin friction resistance and bearing capacity of the pile in frictional soil

This study also divulges that the proportion of pile ultimate resistance (R_t) contributed by skin friction resistance (R_s) and end-bearing capacity (R_b) not only depends on the geometry of the pile alone but also on the angle of internal friction of soil. Figure 3g shows that percentage contribution from the Rb is in increasing order with an increase in ϕ-value while the R_s is reducing as ϕ-value increases. This ascertains that the end-bearing capacity of a pile in a frictional soil depends on soil friction angle. Mathematically, the value of unit end-bearing capacity (q_b) depends largely on N_q for frictional soil, and Equation (20) shows that N_q will increase with an increase in ϕ-value. However, point A in the chart in Figure 3g shows the point at which the contribution of skin friction resistance and end-bearing capacity to the total pile resistance is the same (50% each). The value of the angle of internal friction of soil was observed to be 36° at this point. Hence, for the design assumptions made in this study, 36° is the optimum angle of internal friction.

4.5 Effect of groundwater on pile capacity

The effect of groundwater on pile capacity embedded within a layer below the water table was also studied. This analysis was conducted at an assumed water level depth of 4°m which corresponds to the level of the water table of the site for the selected safe tower project used as a case study. Figure 3h reveals that the pile diameter required to resist the same loads in piles within groundwater is higher than that required where the length of the pile has not reached the water table. Similarly, Figure 3i shows that the pile length resisting the same loads in piles within groundwater is also higher than that required where the length of the pile is in uniform soil with no groundwater. The need for a higher pile size to resist the same loads in soil with groundwater is an indication of a decrease in pile resistance when embedded in soil with groundwater. This decrease in the ultimate resistance of pile in groundwater is attributed to the fact that when soil is submerged its ability to support the load reduces. A cost analysis of this increase in pile length and diameter required showed an average of 19.01% increase in cost for the assumed water level depth.

4.6 Effect of variation in pile costing elements

A cost variation analysis was also carried out to investigate how the overall cost of piling works will be affected by an increase in each of the costing elements. The results show a 25% increase in the costs of labour, concrete and steel will increase the total cost of the pile by 18, 6 and 1% respectively. Meanwhile, an increase in the costing element of the objective function does not affect the geometry of the pile. Rather, it affects the objective function (minimum cost). This indicates that the selection of the pile geometry is independent of the cost. The cost of concrete is a function of total volume. This volume depends on the area and length of the pile that must satisfy both the side and behavioural constraints. The cost of steel is always the minimum cost because a minimum area of steel was provided through cost optimisation. The labour cost varies directly with the volume of concrete. The cost depends on the geometry of piles but not otherwise. However, it is strongly recommended to consider larger diameter and shorter pile to reduce labour cost especially when the piling price considers pile area per length.

4.7 Pile design optimisation for safe tower building

Consequently, the analysis of results obtained from the optimisation of the group pile for the case study project presented in Figure A1 was done and the resulting pile layout from the optimisation is shown in Figure A4. The comparison of the cost between the optimum designs and the conventional design for the chosen case study was conducted based on the same cost calculation procedure and pricelist presented in this study. The results shown in Table 1 revealed that the group optimisation results produced a more economical design with a total cost of £17,885 [26.3% lesser than the conventional method (£23,017)]. This analysis thus established that optimum design will result in a great cost-saving in any large-scale project involving several piles.

5. Conclusion and recommendations

5.1 Conclusions

In this study, an optimisation technique has been employed to demonstrate how to reduce carbon emission through the efficient use of concrete in the construction of concrete piles. The study sustained that optimum design of concrete pile foundation will not only decrease the cost of foundation but also considerably decrease the accumulation of greenhouse gases such as CO_2 in the atmosphere. The CO_2 emission from the concrete production is directly proportional to the cement content used in the concrete mix. Since the minimum quantity of concrete was attained through the design optimisation, then minimum cement will be used and thus result in minimum CO_2 emission. Therefore, this study affirmed that the optimum design of the concrete structure, such as the RC pile presented in this study is one of the vital solutions to limit damaging Earth's climate and the physical environment resulting from high carbon emission.

Compared with other concrete structures, the design of piles can be considered complex due to the uncertainty of soil parameters, the complexity of pile-soil-raft interaction and pile performance prediction. This leads to many constraints that must be satisfied in design.

Ref: S.Figure 3		Bored Pile Sizing from Conventional Design					Bored Pile Sizing from Group Optimisation				
Group /No	Ultimate Load (kN)	Adopted Pile Diameter (mm)	Adopted Pile Length (mm)	Adopted No. of Piles	unit cost (£)	Total cost (£)	Optimised Pile Diameter (mm)	Optimised Pile Length (mm)	Adopted No. of Piles	Unit cost (£)	Total cost (£)
1	903	450	1000	1	509.89	509.89	375.00	9000	1	318.68	318.68
2	1897	450	1000	2	509.89	1019.78	375.00	9000	2	318.68	637.36
3	1921	450	1000	2	509.89	1019.78	375.00	9000	2	318.68	637.36
4	1527	600	1000	1	902.01	902.01	375.00	9000	3	318.68	956.04
5	1551	600	1000	1	902.01	902.01	375.00	9000	3	318.68	956.04
6	2030	450	1000	2	509.89	1019.78	450.00	9000	2	458.91	917.82
7	1937	450	1000	2	509.89	1019.78	450.00	9000	2	458.91	917.82
8	2004	450	1000	2	509.89	1019.78	375.00	9000	3	318.68	956.04
9	2138	450	1000	2	509.89	1019.78	450.00	9000	2	458.91	917.82
10	2487	750	1000	1	1405.57	1405.57	375.00	9000	3	318.68	956.04
11	2470	750	1000	1	1405.57	1405.57	450.00	9000	2	458.91	917.82
12	1647	600	1000	1	902.01	902.01	450.00	9000	3	458.91	1376.73
13	1581	600	1000	1	902.01	902.01	450.00	9000	2	458.91	917.82
14	1935	450	1000	1	509.89	509.89	450.00	9000	2	458.91	917.82
15	2054	750	1000	1	1405.57	1405.57	450.00	9000	2	458.91	917.82
16	2310	450	1000	2	509.89	1019.78	450.00	9000	2	458.91	917.82
17	1991	600	1000	1	902.01	902.01	450.00	9000	2	458.91	917.82
18	2169	750	1000	1	1405.57	1405.57	450.00	9000	2	458.91	917.82
19	672	450	1000	1	509.89	509.89	375.00	9000	1	318.68	318.68
20	7132	750	1000	3	1405.57	4216.71	375.00	9000	5	318.68	1593.40
		TOTAL COST				£23,017	TOTAL COST				£17,885

Table 1. Cost comparison for the case study

However, this study achieved an economical design of pile through the optimum solution using the Generalized Reduced Gradient (GRG) algorithm embedded in Microsoft Excel solver. The results prove that the GRG method in Excel solver is an active, fast, accurate and efficient computer programme to obtain optimum pile design. The main findings of this work were summarised as follows:

(1) The optimum design produced a decrease in the quantity of materials required for specific pile loads.

(2) Pile capacity in sandy soil depends mainly on soil friction angle, and the strength of sandy soils comes mainly from friction between the soil particles.

(3) The total cost of the pile reduces with an increase in the soil friction angle.

(4) High frictional soil has a good bearing capacity and requires smaller piles to carry higher loads.

(5) The study revealed that pile settlement increases with the angle of internal friction of soil.

(6) Young modulus of soil greatly influences pile settlement under specific design load.

(7) The study also concluded that the end-bearing capacity of the pile in frictional soil depends on the frictional angle of soil and that 36° is an optimum angle of internal friction.

(8) The study has produced some design charts that can be used by engineers when designing end-bearing piles in sandy soil for practical projects that fall within the limit of this study assumption. However, a reasonable cost function (Z_c, Z_{st} and Z_{lab}) for the specific region should be adopted to reflect the real construction cost when using these charts.

5.2 Recommendations

Based on the experiences gained from this study, the following recommendations were provided:

(1) Further investigation is required in multiple soil layer systems. The results should be compared to the outcomes of the single soil layer system to investigate the effect of change in the soil in different strata on pile capacity and optimum cost.

(2) More research is also recommended for optimum design of pile foundation in cohesive/frictionless soil.

(3) Further investigation can be carried out on optimisation of different types of pile foundation such as prestressed concrete piles, timber piles and steel tube piles.

(4) Also, different pile geometry like rectangular and hexagonal piles should be studied to investigate the effect of pile shape on its capacity and optimum cost.

(5) The study also recommends an investigation on how to incorporate pile and pile cap parameters in the optimisation statement to get the total optimum cost of foundation works.

(6) Since the use of optimisation techniques shows exciting promises in pile foundation design, further work is needed to explore the many possibilities of carrying out the optimisation through more intricate techniques of optimisation such as using the genetic algorithm to compare the efficiency of these methods.

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Further reading

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Appendix

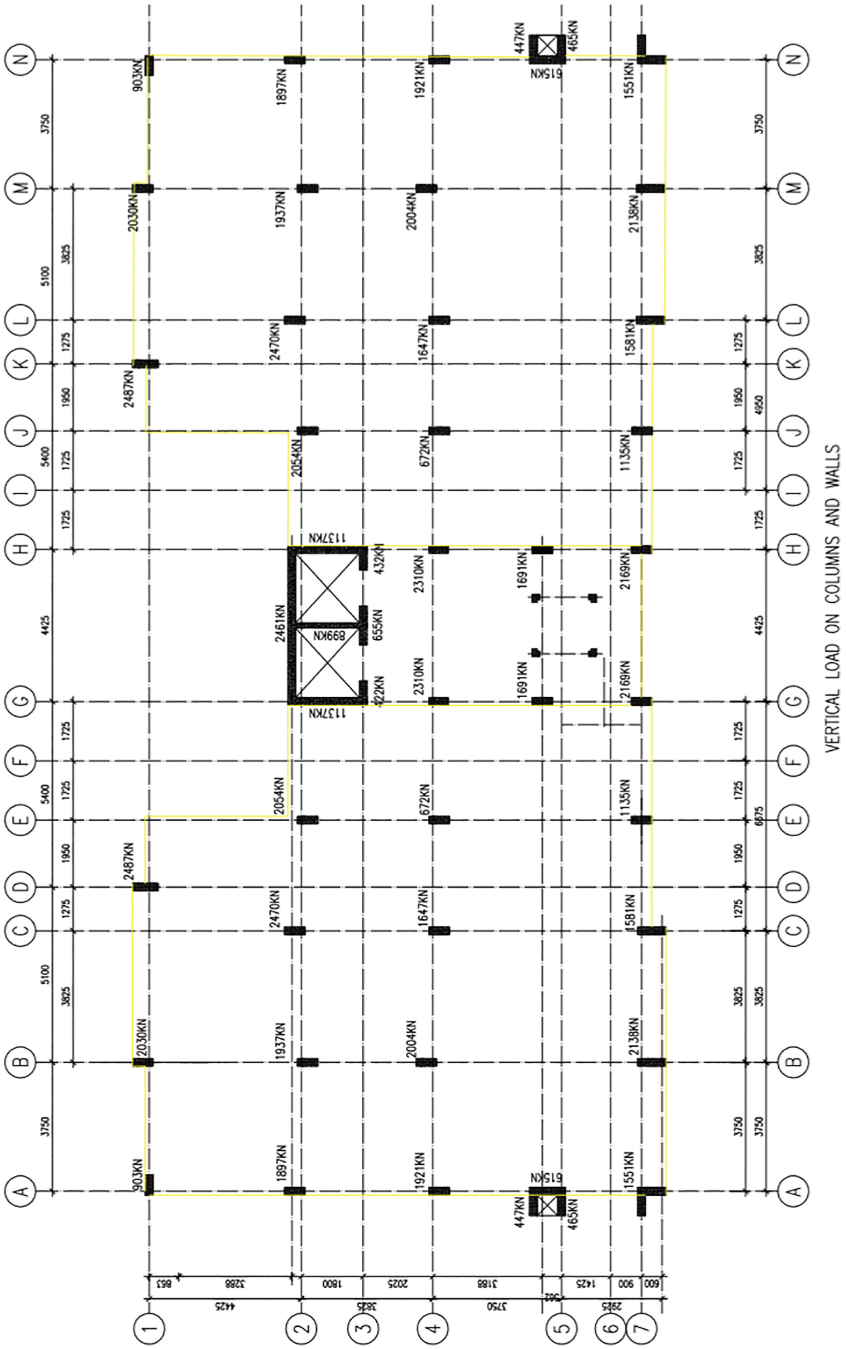

Figure A1.
Vertical loads for pile foundation design (safe tower building)

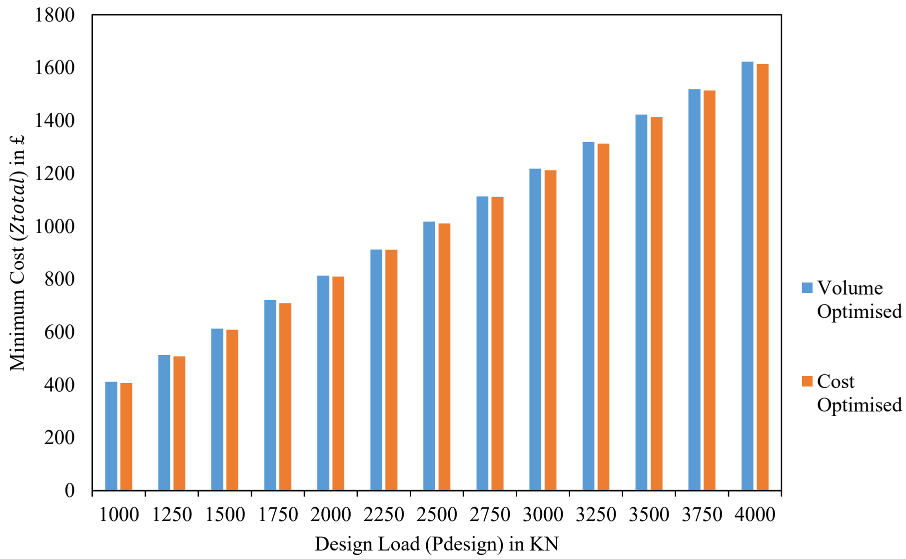

Figure A2.
Minimum cost comparison when volume and cost is optimised for $\phi = 30^{o}$

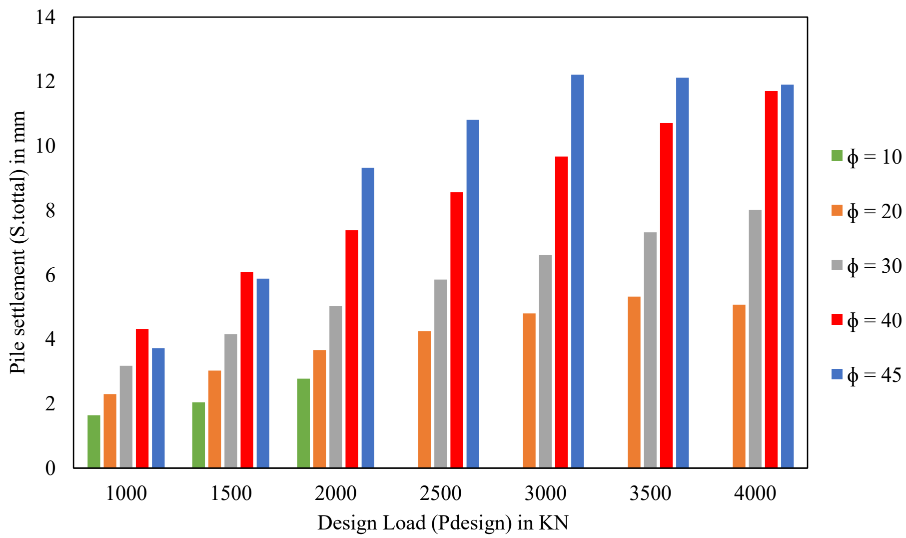

Figure A3.
Pile settlement under applied design load for different soil friction angle

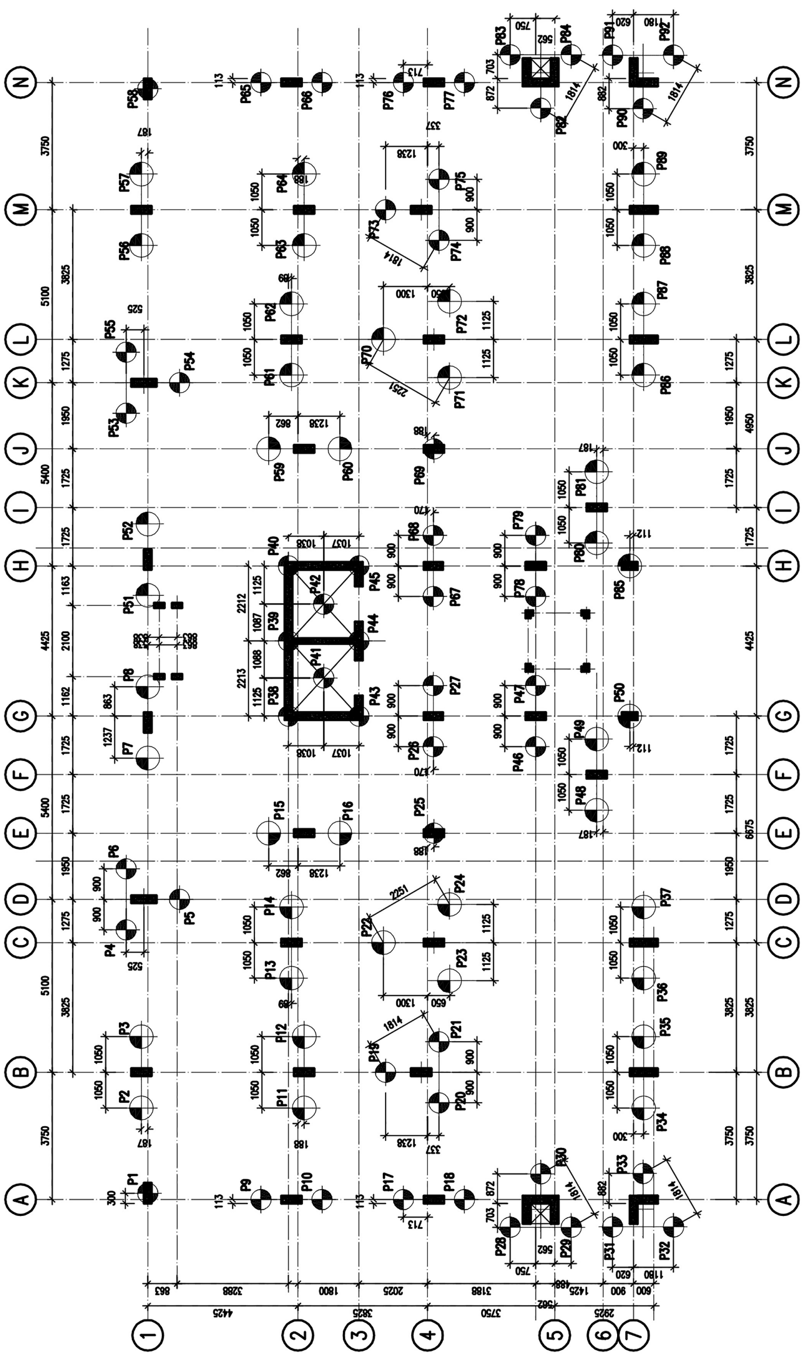

Figure A4.
Resulting pile layout
from the optimisation