Analytical formulation for plain and retrofitted masonry wall under out-of-plane loading

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ABSTRACT: This paper presents an analytical formulation to complement considerable experimental and numerical programs conducted to propose a new timber-based retrofit technique for masonry walls. The formulation is based on the static analysis of an idealised structural scheme adopted in previous experimental tests, and it obtains the bending moment equation which predicts the maximum failure load and its location on both plain and retrofitted masonry walls. Thereafter, the expected failure load in the masonry wall is analytically estimated from the moment of resistance of both the plain and retrofitted walls using ultimate section analysis. The results show that the analytical formulation correctly evaluated the capacity of both the plain and retrofitted masonry wall to within 5% variation. It thus concluded that the developed analytical model can be extended into more complex models and thus fit for a parametric analysis to analyse further the efficiency of the proposed timber masonry retrofit technique.

1 INTRODUCTION

Analytical models and calculations are basic skills employed by engineers as a useful complement or alternative to experimental tests and numerical models. This is because full experimental investigation on structural elements, particularly masonry is always tedious and expensive. Therefore, this paper presents an analytical formulation to complement considerable experimental and numerical programs conducted to propose a new timber-based retrofit technique for masonry walls (Dauda et.al, 2021; Iuorio et.al, 2021). Masonry is a very complex, heterogenous and composite structure that possess non-linear mechanical properties. The non-linearity in the mechanical behaviour of masonry causes high uncertainty in analysing its structural response (Lourenco and Silva, 2020; Ismail and Ingham, 2016). As such, the analytical prediction of masonry strength and responses is a very complex subject area that is attracting interest from various researchers in the field.

Therefore, this paper focuses on an analytical formulation for plain and retrofitted masonry walls under out-of-plane loading. The formulation is based on the static analysis of the idealised structural schemeadopted in the experimental test setup. The formulation obtains the bending moment equation which predicts the maximum failure load and its location on both plain and retrofitted masonry walls. The results of the analytical formulation were compared against the experimental results obtained from Iuorio et.al (2021).

The paper was articulated as follows. In section 2, the analytical formulation for both plain and retrofitted masonry walls was presented. Section 3 presents a brief comparison of the analytical result with the experimental results. Finally, the conclusion to the study is presented in section 4.

2 ANALYTICAL PREDICTION OF MASONRY WALL STRENGTH

An analytical model in form of a simplified structural scheme (Fig. 1) which accounts for a horizontal restraint due to the friction at the base of the four-point bending experimental test setup (Fig. 2) was used in this study to obtain the strength of masonry wall as subsequently described.

Referring to figure 1, the support reaction at the top and bottom of the masonry wall (R_A and R_B) are estimated from static equilibrium equations following equation 1 to 4.

$$R_A + R_B = P + F_\mu \qquad \text{Equation 1}$$

$$R_A = \frac{P}{2} - \left(F_\mu * \frac{X_3}{X_1 + 2X_2}\right) \qquad \text{Equation 2}$$

$$R_B = \frac{P}{2} + \left(F_\mu * \frac{X_1 + 2X_2 + X_3}{X_1 + 2X_2}\right) \qquad \text{Equation 3}$$

$$F_\mu = \mu \times \gamma_{mw} \qquad \text{Equation 4}$$

Where:

F_μ: is the resultant horizontal force due to friction at the base of the specimen.

μ: coefficient of friction between steel roller and plate at the base of the wall (0.8)

γ_{mw}: unitary weight of wall calculated using the density of brick unit (2200kg/m^3)

Meanwhile, in order to estimate the bending moment equation which will predict the maximum failure load at any location (X_i) within the inner bearing of the masonry wall, a section Y-Y is considered, and the moment equation is derived using equation 5 or 6.

$$M = \frac{P}{2}(X_i) - R_A(X_2 + X_i) \qquad \text{Equation 5}$$

$$M = \frac{P}{2}(X_2) - \left(F_\mu * \frac{(X_2 + X_i) * X_3}{X_1 + 2X_2}\right) \qquad \text{Equation 6}$$

Thereafter, the expected failure load in the masonry wall was estimated analytically by considering the moment of resistance of both the plain and retrofitted walls using ultimate section analysis in section 2.1 and section 2.2.

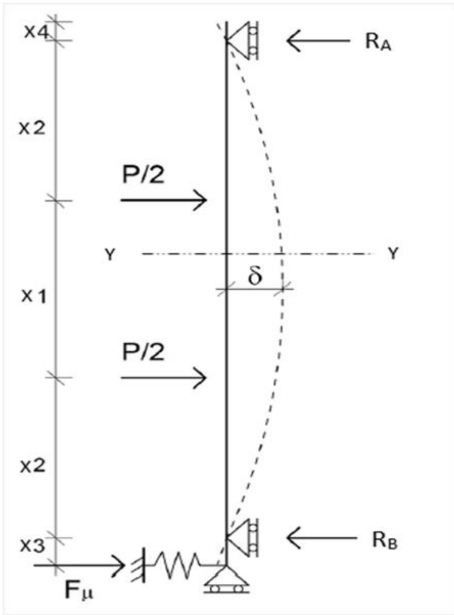

Figure 1. Idealised structure scheme for four point bending test

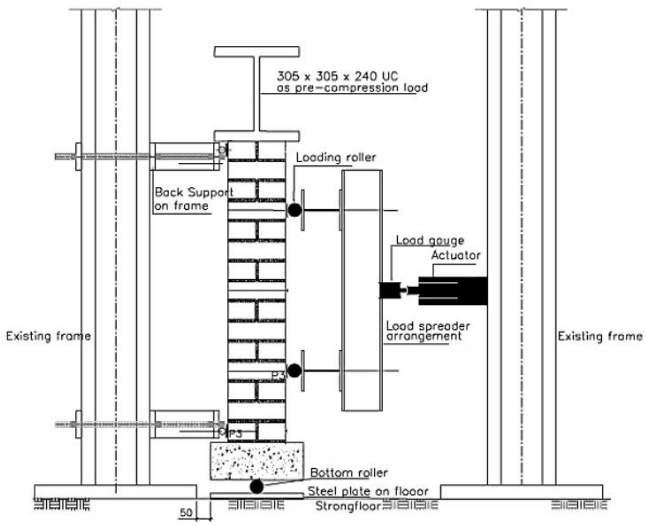

Figure 2. Test set up for four point bending test

2.1 *Moment of Resistance (Mrd) of Plain Wall*

Using the provisions from EC6 and inferences from Gattesco and Boem (2017), the moment of resistance of plain masonry panel can be estimated following the equations 7 - 9.

$$M_{rd(mw)} = \left(\frac{f_{xk}}{f_s} + \sigma_d\right) Z \qquad \text{Equation 7}$$

$$Z = \frac{bd^2}{6} \qquad \text{Equation 8}$$

$$M_{rd(mw)} = (f_{xk} + \sigma_d) * \frac{bd^2}{6} \qquad \text{Equation 9}$$

Thereafter, combining both equations 6 and 9 together as shown in equations 10 and 11 will give the estimated maximum load (P) that causes failure of the plain masonry wall.

$M = M_{rd}$ Equation 10

$\frac{P}{2}(X_2) - \left(F_{\mu} * \frac{(X_2+X_i)*X_3}{X_1+2X_2}\right) = (f_{xk} + \sigma_d) * \frac{bd^2}{6}$

Equation 11

Where;

f_{xk}: Flexural strength of masonry wall

f_s: is the factor of safety; neglected to compare with the experimental result

σ_d: is the imposed vertical load per unit area estimated to be 0.0125N/mm^2 i.e. 3000 / (215*1115) from the load used during the experimental test in Iuorio et.al (2021).

Z: is the section modulus of the plan shape of the specimen.

2.2 *Moment of Resistance (Mrd) of Retrofitted Wall*

2.2.1 *One Side Retrofitted Wall (1SRW)*

A composite section comprising a masonry part and OSB timber panel part is developed in the retrofitting technique. Analytically, the true moment capacity of the composite section subjected to positive bending can be closely approximated by assuming that either the tensile strength is attained in the OSB panel, or the masonry section is stressed to $0.85f_{ck}$ (Dauda et.al, 2020; Chen and He, 2017). Hence, the moment of the resistance of the composite timber-masonry retrofitted wall can be estimated from equilibrium by considering the stress block at the ultimate limit state as shown in figure 3.

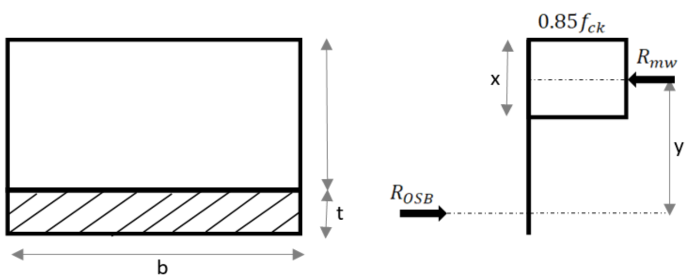

Figure 3. Ultimate section analysis for one side retrofitted wall

Considering figure 3, the resistance of the masonry section (R_{mw}) and the resistance of the OSB timber section (R_{OSB}) is estimated using equation 12 and 13 respectively.

$R_{mw} = 0.85 * f_{ck} * b * d$ Equation 12

$R_{OSB} = f_y * b * t$ Equation 13

where:
f_{ck}: is the compressive strength of masonry;
f_y: is the tensile strength of the OSB panel.

Since the resistance of the OSB panel is less than that of the masonry section, the neutral axis of the composite section is within the masonry section (Drysdale et.al, 1993) and the depth of the masonry compression stress block (x) in figure 3 is calculated from equation 14.

$0.85 * f_{ck} * b * x = f_y * b * t$ Equation 14

Still with reference to figure 3, the moment of resistance of the composite section ($M_{rd(mwOSB)}$) for full shear connection is calculated using equation 15.

$M_{rd(mwOSB)} = R_{OSB} * y$ Equation 15

where y is the lever arm to the centre of the OSB timber panel estimated from figure 3.
Finally, to analytically predict the maximum load (P) that will cause the failure of the retrofitted wall, equations 6 and 15 were combined as shown in equation 16. This is also similar to the same concept used to analyse the failure load of a plain masonry wall in section 2.1

$\frac{P}{2}(X_2) - \left(F_{\mu} * \frac{(X_2+X_i)*X_3}{X_1+2X_2}\right) = M_{rd(mwOSBC)}$

Equation 16

2.2.2 *Two Side Retrofitted Wall (2SRW)*

For the two sides application shown in figure 4, the compression section comprising the OSB and masonry is converted to an equivalent section because of the composition with two parts having different stiffness. An equivalent thickness of the OSB with respect to the masonry is obtained by multiplying OSB thickness (t_{osb}) by a factor (n) which is the ratio of Young's modulus of the OSB to the masonry (Figure 4.). After that, the neutral axis of the retrofitted section is calculated from the new thickness of the equivalent section. The new properties of the equivalent section were then used in the equation of bending (Equation 15) to determine the flexural strength of the retrofitted specimens.

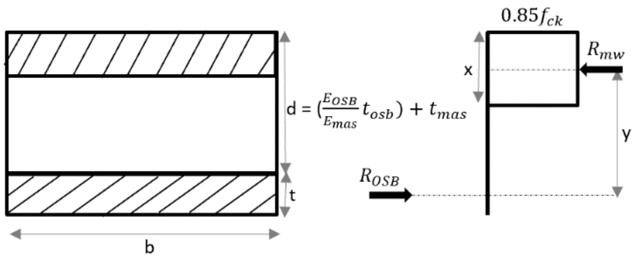

Figure 4. Ultimate section analysis for two sides retrofitted wall

Similar to the one-sided application, the resistance of the OSB panel in the tensile section is less than that of the masonry and OSB on the compression face, therefore the neutral axis of the composite section is within the compression face (i.e the masonry compression section) and the depth of the masonry compression stress block (x) in figure 4 is also calculated and used in equation 14.

3 RESULTS AND DISCUSSION

In this section, the result of the analytical prediction formulated compared to the obtained result from the experiment (Iuorio et.al, 2021) is discussed for both the plain and retrofitted masonry wall. Table 1 - 3 presents the failure load obtained analytically alongside the actual damage load obtained from the test for both plain and retrofitted models respectively.

Using the geometry of the wall and the mechanical properties of the constituent materials, the following parameters were used in the appropriate equations in section 2.

$$F_{\mu} = 4704N, f_{xk} = 0.54\, N/mm^2$$
$$b = 1115mm, d = 215mm$$
$$X_1 = 600mm, X_2 = 245mm, X_3 = 32.5mm$$
X_i*: varies for different specimens*

Table 1. Results for Plain Masonry Wall

Specimen	PW1115-1	PW1115-2
Xi (mm)	257.5	157.5
M_{rd} (Nmm)	4746056	4746056
F_{μ} (Nmm)	4704	4704
X1 (mm)	600	600
X2 (mm)	245	245
X3(mm)	32.5	32.5
Analytical Result (N)	39300	39200
Experimental Result (N)	39700	38300
Difference (%)	1	2

Table 2. Results for One Side Retrofitted Masonry Wall

Specimen	1SRW1115-1	1SRW1115-2
Xi (mm)	157.5	92.5
M_{rd} (Nmm)	13445820	13445820
F_{μ} (Nmm)	5210	5210
X1 (mm)	600	600
X2 (mm)	245	245
X3(mm)	32.5	32.5
Analytical Result (N)	110270	110190
Experimental Result (N)	116400	116400
Difference (%)	5	2

Table 2. Results for Two Side Retrofitted Masonry Wall

Specimen	2SRW1115-1	2SRW1115-2
Xi (mm)	322.5	257.5
M_{rd} (Nmm)	14047920	14047920
F_{μ} (Nmm)	5720	5720
X1 (mm)	600	600
X2 (mm)	245	245
X3(mm)	32.5	32.5
Analytical Result (N)	115400	115500
Experimental Result (N)	119500	121700
Difference (%)	3	5

4 CONCLUSION

The study presented in this paper has demonstrated that the results of experimental tests conducted on four bending tests (Iuorio et.al, 2021) can be predicted analytically. The analytical formulation was based on using the stress state of the generic cross-section of masonry specimen. The formulation assumed the combined effect of the axial force, due to the masonry self-weight, and the bending action induced by the two horizontal loads.

The model predicted the peak load and the corresponding failure to within less than 5% of the average results obtained from the test. The comparative analysis of both the analytical and experimental results shows that the formulated equations can adequately capture the behaviour and failure of both the plain and retrofitted masonry walls. This indicates that the analytical model can be employed to carry out a parametric study to investigate the performance of the proposed retrofit technique further.

The main limitation of this model is the exclusion of the anchor connection with the assumption that there is a full shear connection between the masonry wall and OSB. Hence, a detailed analytical model and parametric study to assess the model capability to simulate URM walls retrofitted with different OSB panel thicknesses and connection layout is recommended for future study.

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